# Non-perturbative study of Yang-Mills theory with four supercharges in two dimensions


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With Raghav G. Jha, Anosh Joseph, and David Schaich

# Quick RECAP

Presented preliminary analysis in **Lattice 2021**.
$\Rightarrow$ Some remarks from the talk:

**Scalars behaviour**
Existence of bound state at finite temperature for $U(N)$ with $N = 2, 4, 8, 12$.
arXiv:2109.01001 [hep-lat]   NSD, Jha, Joseph, Schaich

**Comparison with $16$ supercharge theory**
Theory looks to be in different universality class to maximal theory.

**Not discussed**
Possible 'Spatial Deconfinement' transition.

# This talk

- **Overview** of four supercharge theory on **Lattice**.
- **Comparison** with maximal theory in **different coupling** regimes.
- Signature of **'Spatial Deconfinement'** transition and its possible **order**.
- **Phase structure on rectangular torus**.

# Two-dimensional $\mathcal{N} = (2,2)$ SYM

# Two-dimensional $\mathcal{N} = (2, 2)$ SYM

Constructed from dimensional reduction of four dimensional theory.

$$\mathcal{N} = 1,\ d = 4 \quad \rightarrow \quad \mathcal{N} = (2,2),\ d = 2$$

- Not a "**maximal**" theory.
- No holographic dual "**exists**".
- Regularised on lattice using "**twisting**".
  Phys. Rep. **484** (2009) 71-130
  *Catterall, Kaplan, Ünsal*

**Maximal Supersymmetric theories on Lattice talks:**

*Goksu Toga*: Now TD-I

*Angel Sherletov*: Monday-5:10 pm

*David Schaich*: Monday-5:30 pm

*Arpith Kumar*: Wednesday-4:50 pm

# Two-dimensional $\mathcal{N} = (2,2)$ SYM

### Continuum Action

$$S = \frac{N}{4\lambda} \mathcal{Q} \int d^2x \, \text{Tr} \left( \chi_{ab} \mathcal{F}_{ab} + \eta \left[ \overline{\mathcal{D}}_a, \mathcal{D}_a \right] - \frac{1}{2} \eta d \right)$$

### After integrating out auxiliary field

$$S = \frac{N}{4\lambda} \int d^2x \, \text{Tr} \left( -\overline{\mathcal{F}}_{ab} \mathcal{F}_{ab} + \frac{1}{2} \left[ \overline{\mathcal{D}}_a, \mathcal{D}_a \right]^2 - \chi_{ab} \mathcal{D}_{[a} \, \psi_{\, b]} - \eta \overline{\mathcal{D}}_a \psi_a \right)$$

# Two-dimensional $\mathcal{N} = (2,2)$ SYM

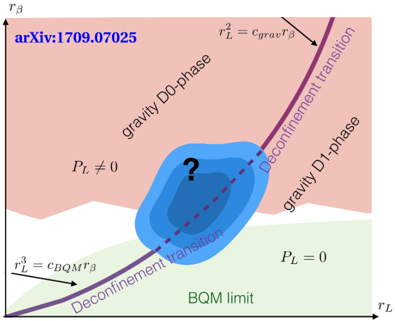
$$S = \frac{N}{4\lambda} \int d^2x \, \text{Tr} \left( -\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2} \left[ \overline{\mathcal{D}}_a, \mathcal{D}_a \right]^2 - \chi_{ab}\mathcal{D}_{[a}\,\psi_{\,b]} - \eta\overline{\mathcal{D}}_a\psi_a \right)$$

- Using geometrical discretization $\rightarrow$ theory lives on 2d lattice.
  JHEP **11** (2004) 006
  *Catterall*
- To control flat directions, scalar potential term added to discretized action.
  JHEP **11** (2012) 072
  *Catterall, Damgaard, DeGrand, Galvez, Mehta*
- Discretization used and all the observables studied can be accessed via publicly available software github.com/daschaich/susy.

# Comparison with Two-dimensional $\mathcal{N} = (8, 8)$ SYM

# Two-dimensional $\mathcal{N} = (8, 8)$ SYM

- At low temperature and large $N \Rightarrow$ dual to type IIB supergravity.




PRD **97**, 086020 (2018)
*Catterall, Jha, Schaich, Wiseman*

## Maximal theory prediction from gravity dual

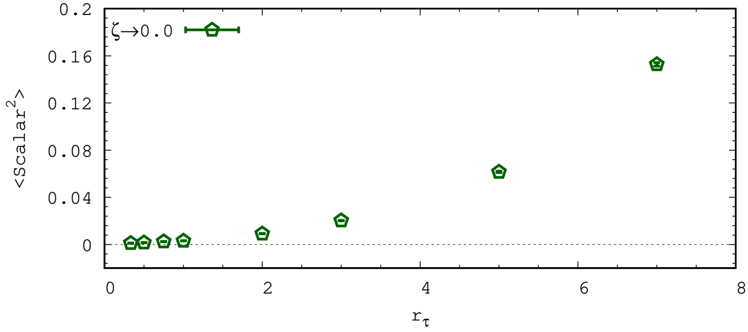
- Scalars behave as: Tr$(X^2) \propto t$.
  JHEP **07** (2013) 101 *Wiseman*
- Energy density $\propto t^2$ (for $t > 1$),
  $\propto t^3$ (for $t < 1$).
  JHEP **07** (2013) 101 *Wiseman*
- First order GL Phase transition.
  PRL **70**, 2837 (1993) *Gregory, Laflamme*

# Back to Target Theory

$\Rightarrow$ Lattice simulations for four supercharge theory $\Leftarrow$

- Worked with finite mass deformation parameter $\mu$,
  $\mu = \zeta \frac{r_\tau}{N_t} = \zeta\sqrt{\lambda}a$, $r_\tau = 1/t$, $\quad 0.33 \leq r_\tau \leq 7.0$, $\zeta \in (0.2, 0.3, 0.4, 0.5)$ .
- Different Lattice aspect ratios $\alpha$,
  $\alpha = \frac{N_x}{N_\tau} = \frac{r_x}{r_\tau} \qquad \alpha \in (0.5, 1.0, 1.5, 2.0)$.
- Different gauge groups, $\quad 2 \leq N \leq 20$
- Anti-periodic boundary conditions for fermions along temporal direction.

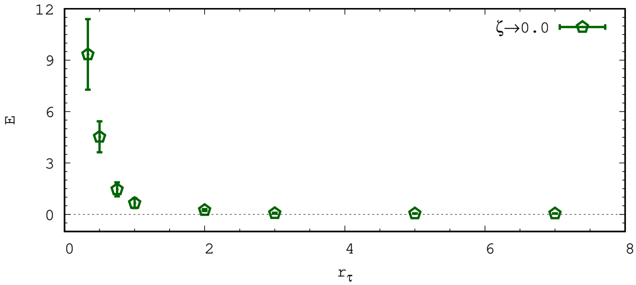
# Scalar behaviour




- Scalar$^2$ $\leftrightarrow$ Tr$(X)^2$
- $24 \times 24$ lattice, $N = 12$.

- $r_\tau > 1 \rightarrow r_\tau^3$ behaviour, Maximal case $\rightarrow 1/r_\tau$.
- $r_\tau < 1 \rightarrow r_\tau$ behaviour, Maximal case ??

# Energy density

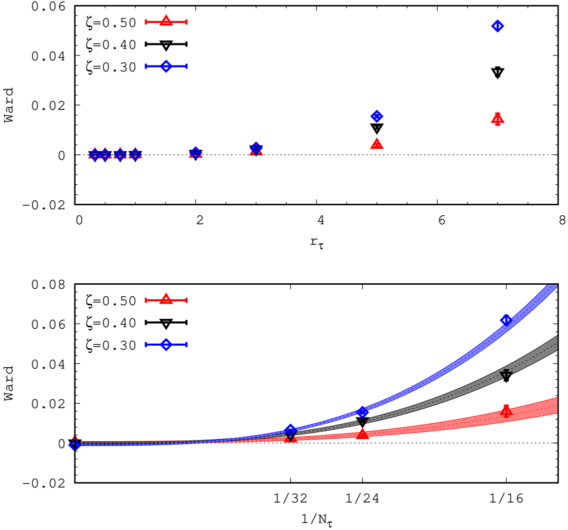



- $E = \frac{3}{\lambda_{lat}}\left(1 - \frac{2}{3N^2}S_B\right)$
- $24 \times 24$ lattice, $N = 12$.

- $r_\tau > 1 \rightarrow r_\tau^0$ behaviour, Maximal case $\rightarrow 1/r_\tau^3$.
- $r_\tau < 1 \rightarrow 1/r_\tau^2$ behaviour, Maximal case $\rightarrow 1/r_\tau^2$.

- Vanishing energy density at zero temperature $\rightarrow$ Preserved SUSY.
  PRD **80**, 065014 (2009) *Hanada, Kanamori*
  PRD **97**, 054504 (2018) *Catterall, Jha, Joseph*

# Ward Identity




**Ward Identity:**
$\mathcal{Q} \sum_a \left( \eta \mathcal{U}_a \bar{\mathcal{U}}_a \right)$

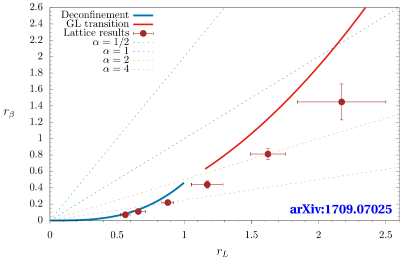
- At larger temperatures ($r_\tau < 1$), ward identity satisfied.
- At smaller temperatures ($r_\tau > 1$), satisfied at larger volume.

$24 \times 24$ lattice, $N = 12$
Bottom left plot with $r_\tau = 5.0$.

# ‘Spatial deconfinement’ in two-dimensional $\mathcal{N} = (2,2)$ SYM

# Spatial deconfinement

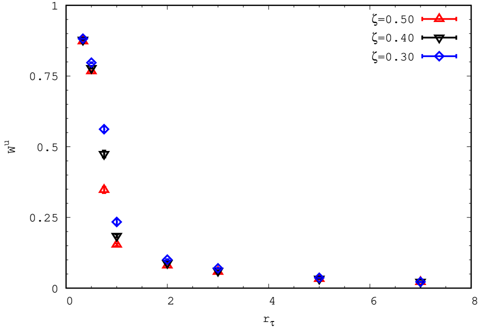
- Four supercharge theory: so far .....
  - Preserved SUSY.
  - Different behaviour compared with maximal case.
  - What about deconfinement transition? which exists in sixteen supercharge theory.

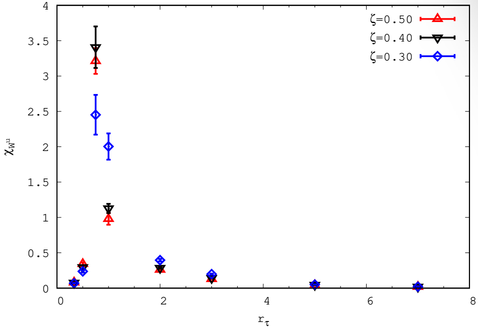



PRD **97**, 086020 (2018)

*Catterall, Jha, Schaich, Wiseman*

# Spatial deconfinement - Signal

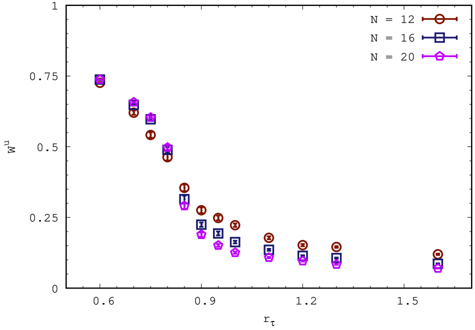


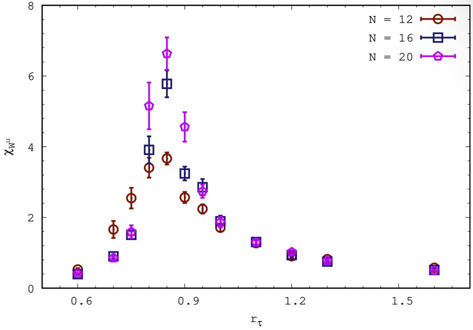




- Spatial wilson lines and its susceptibilty as order parameter for deconfinement transition.
- $24 \times 24$ lattice, $N = 12$.

- Transition around $r_\tau = 1.0$.
- Slight $\zeta$ dependence.

# Spatial deconfinement - Order

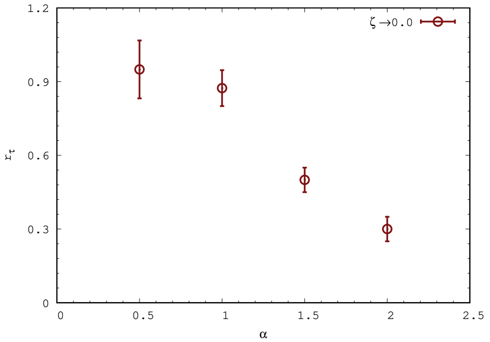



- Spatial wilson lines and its susceptibilty for different $N$ values.
- $12 \times 12$ lattice, $\zeta = 0.30$.

- Critical $r_\tau$ independent of $N$.
- Hints of second-order transition.
  PRL **113**, 091603 (2014)
  *Azuma, Morita, Takeuchi*

# Phase transition vs Lattice aspect ratio




- $N = 12$, Lattices used $12 \times 24, 12 \times 12, 24 \times 16, 24 \times 12$.
- $r_\tau$(critical) has $\zeta$ dependence for $\alpha \leq 1$.

- Spatial deconfinement transition similar to maximal theory but restricted only in weaker coupling regime.

# Conclusions

**Spatial deconfinement**
Spatial deconfinement phase transition observed in this theory with different lattice volumes.

**Weak coupling behaviour**
Similar to maximal theory,
with different normalizations

- Phase transition observed.
- Energy density behaviour same.

**Strong coupling behaviour**
Different from maximal theory,

- No Phase transition.
- Scalars behaviour different.
- Energy density behaviour different.

**Open question**
Holographic dual to two-dimensional $\mathcal{N} = (2,2)$ SYM - - - - ???

# Thanks for your attention

## Resources

## Follow up

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