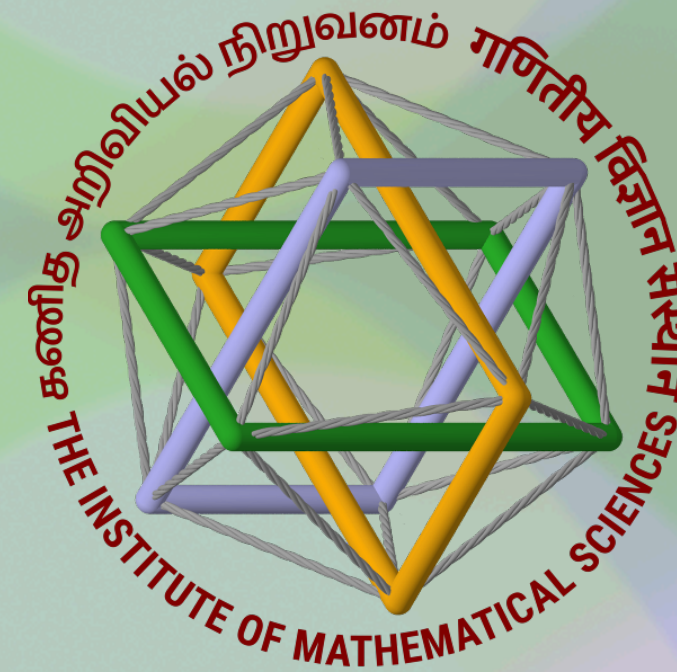


Talk slides will be available at  
<https://navdeep-dhindsa.github.io/>

[navdeep.s.dhindsa@gmail.com](mailto:navdeep.s.dhindsa@gmail.com)

**Navdeep Singh Dhindsa 15/07/2024**

# Insights into Dibaryon Interactions in the Heavy Quark Sector

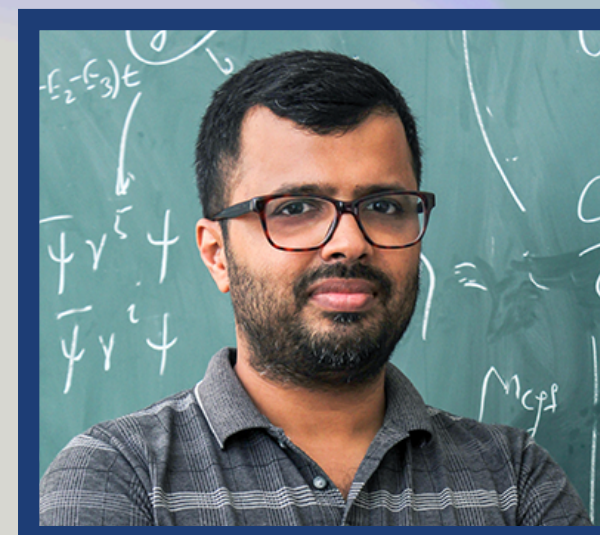


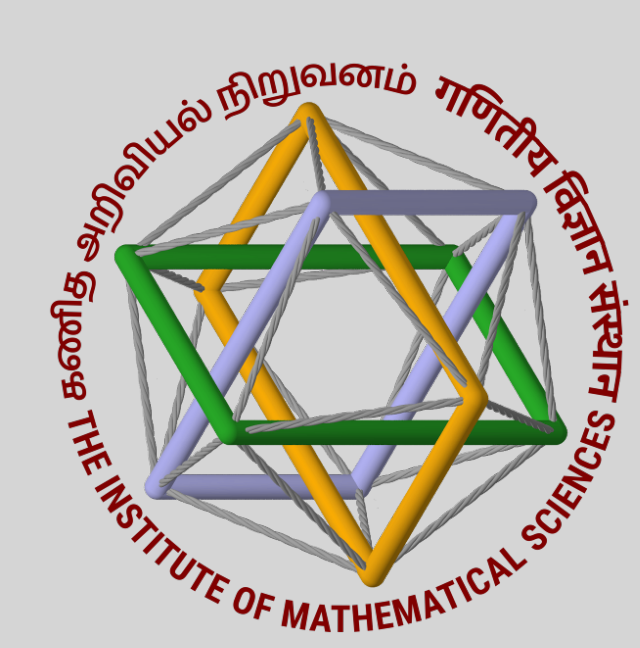
## Funding resources



**Work in collaboration with**

**M. Padmanath (IMSc Chennai) and Nilmani Mathur (TIFR Mumbai)**





# Deuteron

- Deuterium destroyed in interior of stars faster than it is produced.
- All deuterium found in nature has origin from Big Bang nucleosynthesis. (Temperature hot enough to produce it but not hot enough to produce byproduct or get destroyed)
- Deuterium bottleneck after big bang and then only 20 minutes for nucleosynthesis.

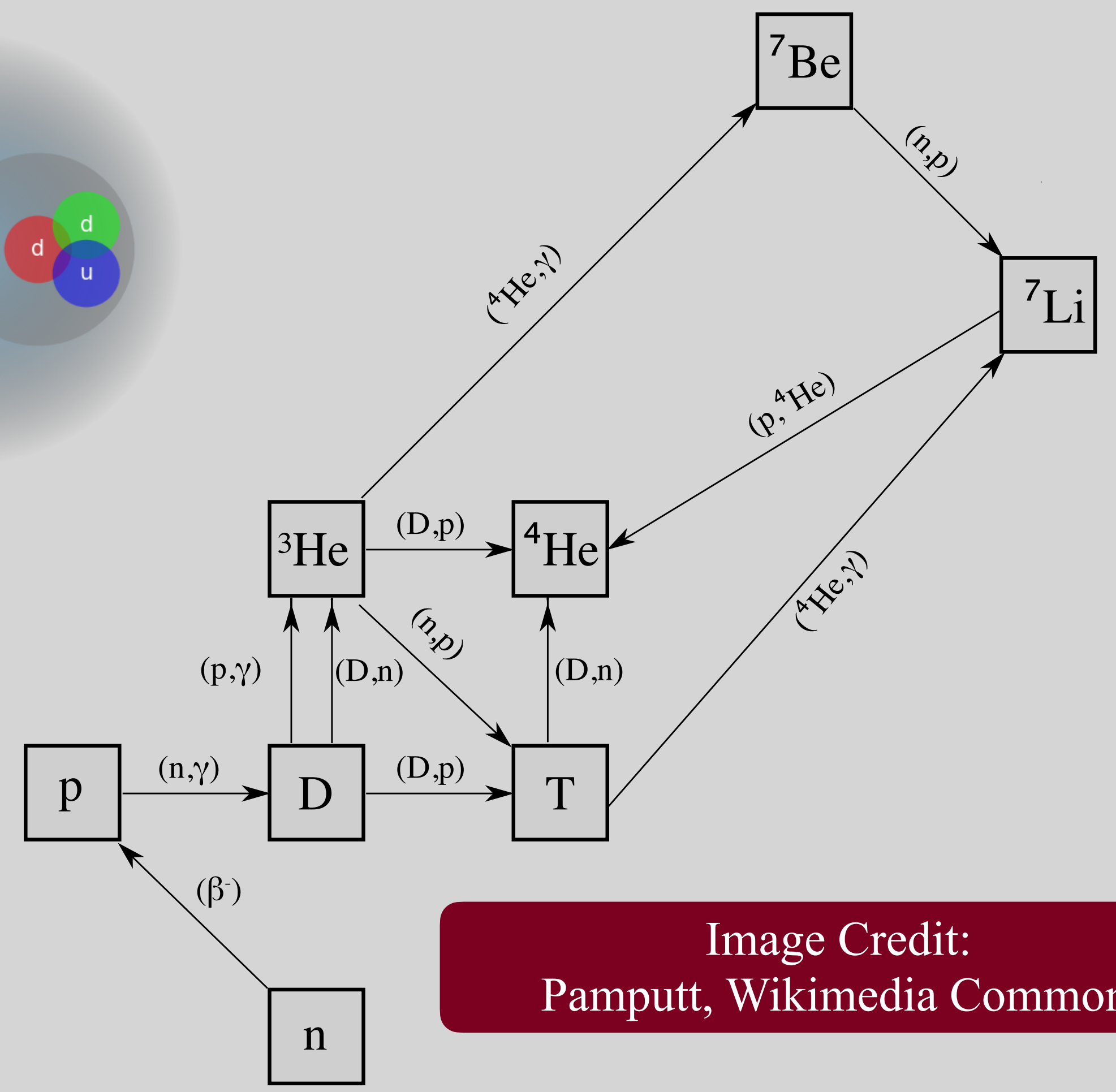
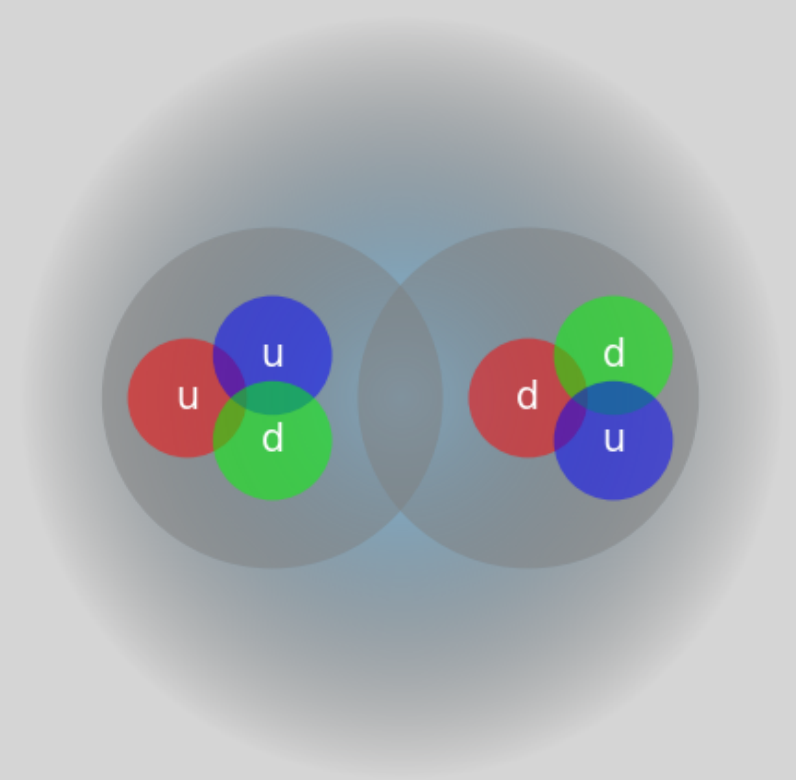
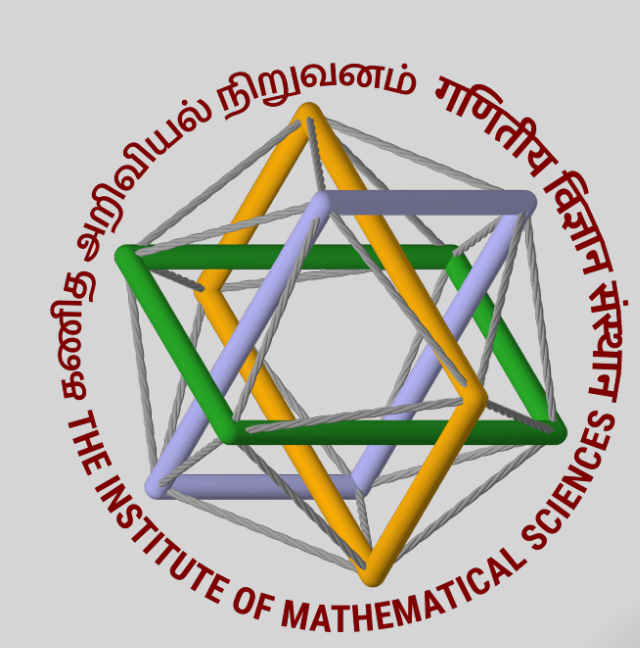


Image Credit:  
Pamputt, Wikimedia Commons

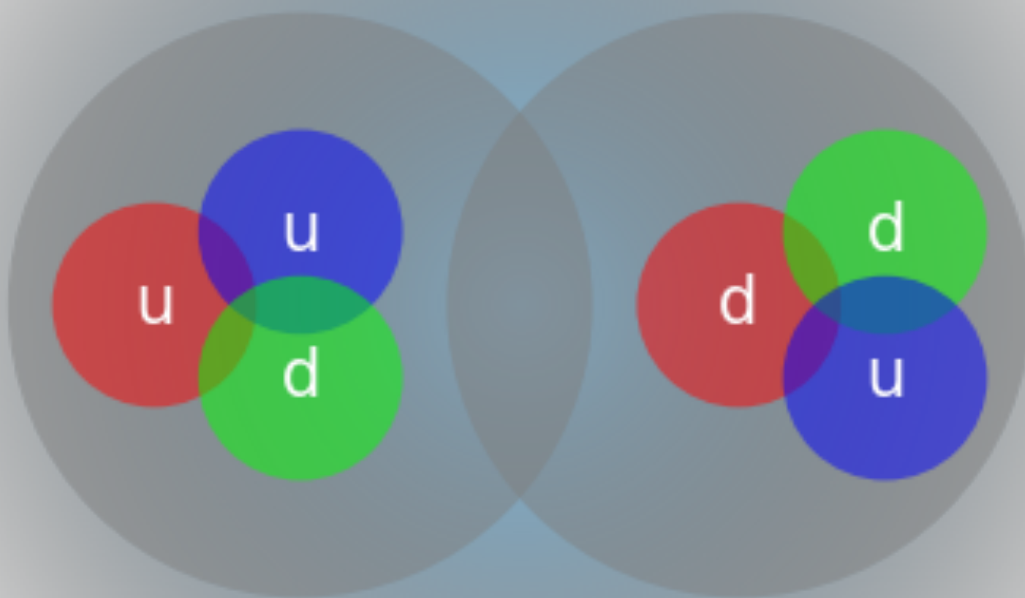
Based on theory of strong interactions, we cannot rule out more dibaryons in nature.

Dineutron? Diproton?  
NN scattering experiments indicates absence of bound state





# Deuteron Dibaryon



- Many predictions of various dibaryon states but failed experimental checks
- Recent renewal in interest due to discoveries of complex quark systems (not just baryons/mesons)
- Experimental evidence of existence of  $d^*$

Any object with Baryon number 2  
Composed of six valence quarks  
Can be molecular or compact

- Discovered around a century back
- Proton (uud) - Neutron (udd) bound state
- Binding Energy = 2.2 MeV

1932

1950's

2010's

Clement  
PPNP (2016)

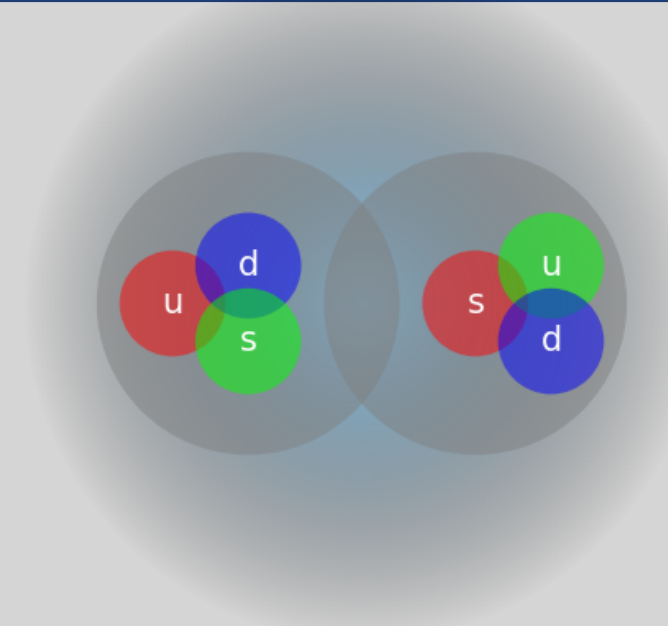


# Hyperon-Nucleon

Experimental results indicate:

- $\Lambda N$  interaction is attractive, though less than  $NN$ , there is no strange deuteron.
- $\Sigma N$  interaction even weaker than  $\Lambda N$ .
- $\Lambda\Lambda$  does not rule out bound system.

- \* NAGARA event - constraint on binding energy
- \* Dedicated experiments for H dibaryon indicates existence unlikely but its existence not ruled out yet.



\* Jaffe prediction of dihyperons

Jaffe  
PRL 138 (1977) 195

\* Theoretical calculations vary from very deep bound (even more than Jaffe's prediction) to unbound.

\* Lattice QCD - gives bound result (8 MeV) - Large pion mass used (discussed later).

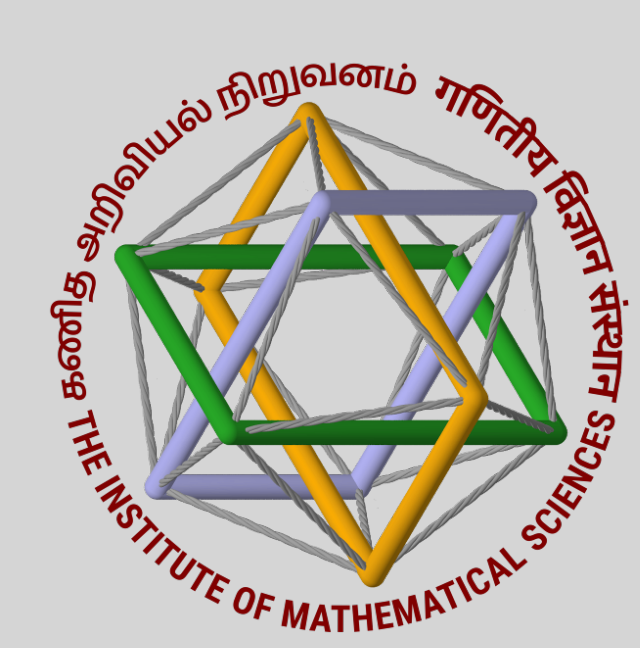
Beane et al.  
PRL 106(2011) 162001

\* Chiral effective field theory calculation - smaller binding energy.

Talk by Green  
Santa Fe Workshop 2023

Inoue et al.  
PRL 106(2011) 162002

Haidenbauer and Meissner  
PLB 206(2011) 100



# $d^*(2380)$

Dyson and Xuong  
PRL 13 (1964) 815

WASA @ COSY collaboration  
SAID Data Analysis Center  
PRL 112 (2014) 202301

1964 - Prediction of possible bound states

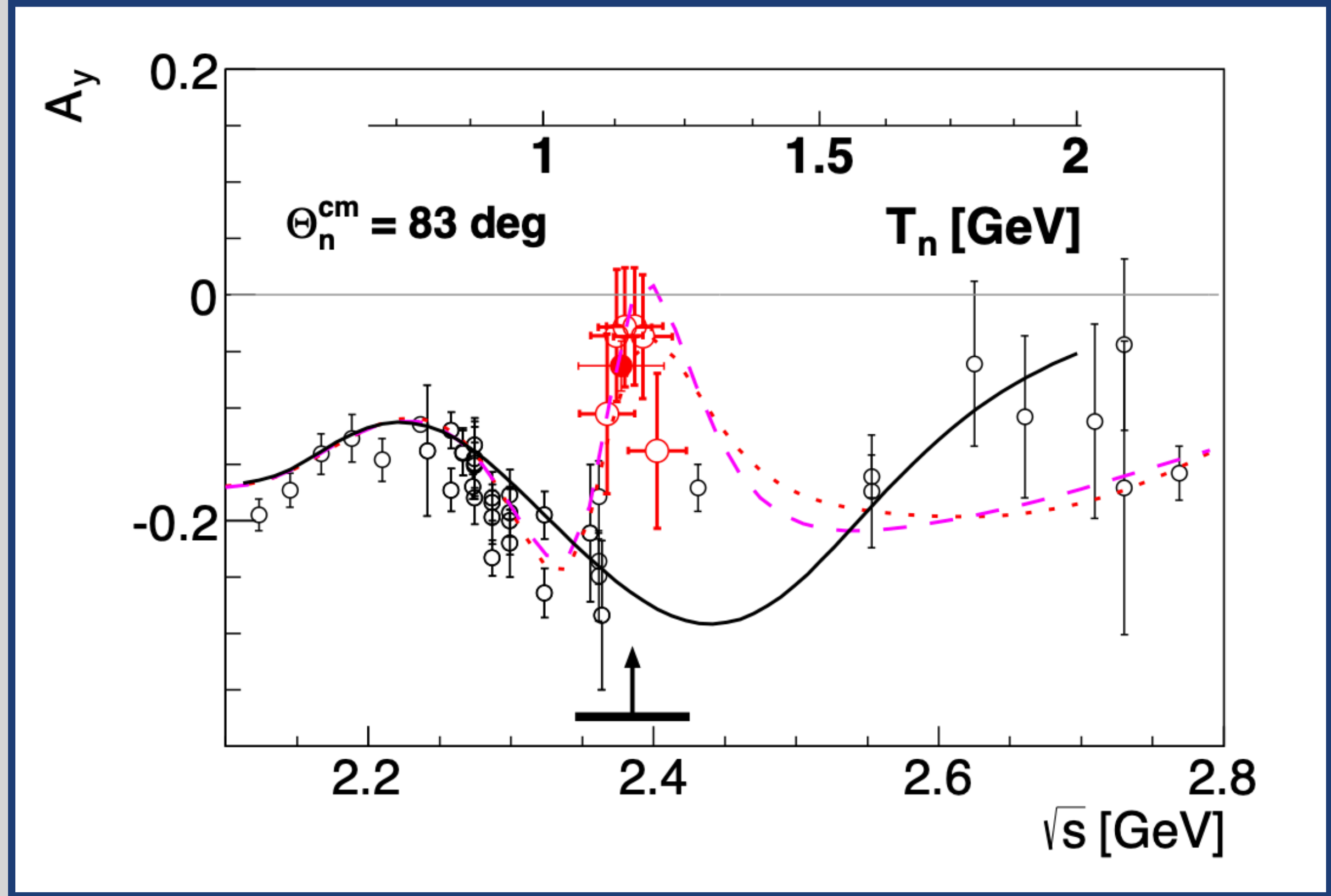
They predicted mass of  $D_{03}$  (close to  $d^*$  which was found later)

## Bremsstrahlung measurements

Kamae and Fujita  
PRL 38 (1977) 471

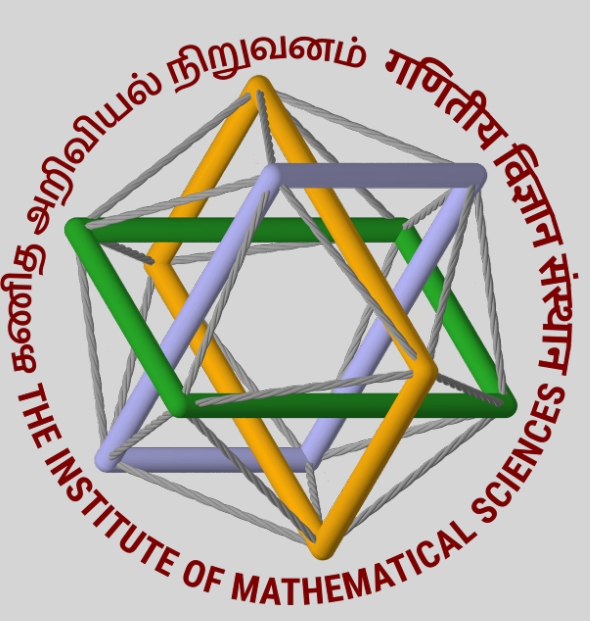
“An inevitable non-strange dibaryon”

Goldman et al.  
PRC 39 (1989) 1889

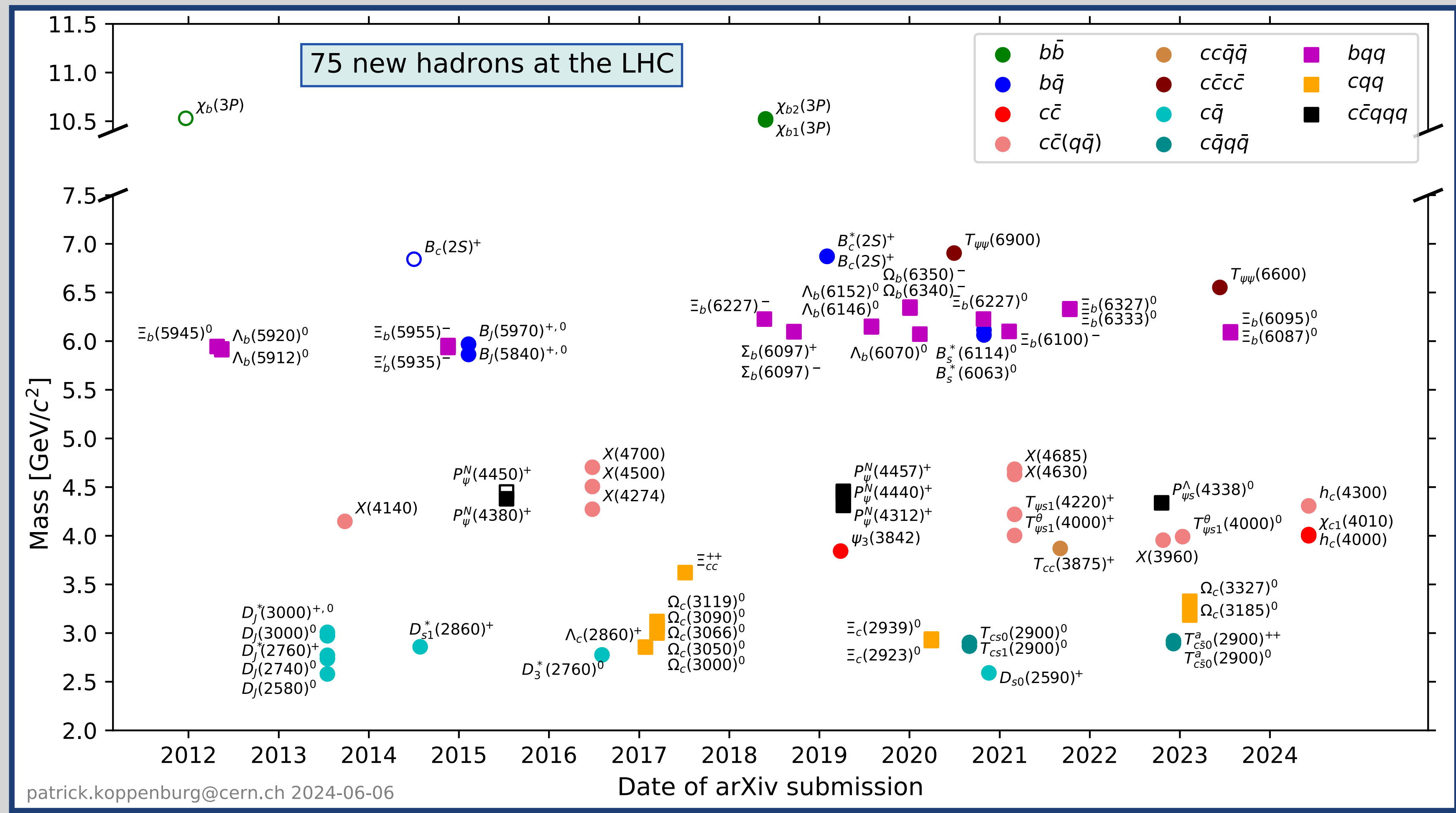
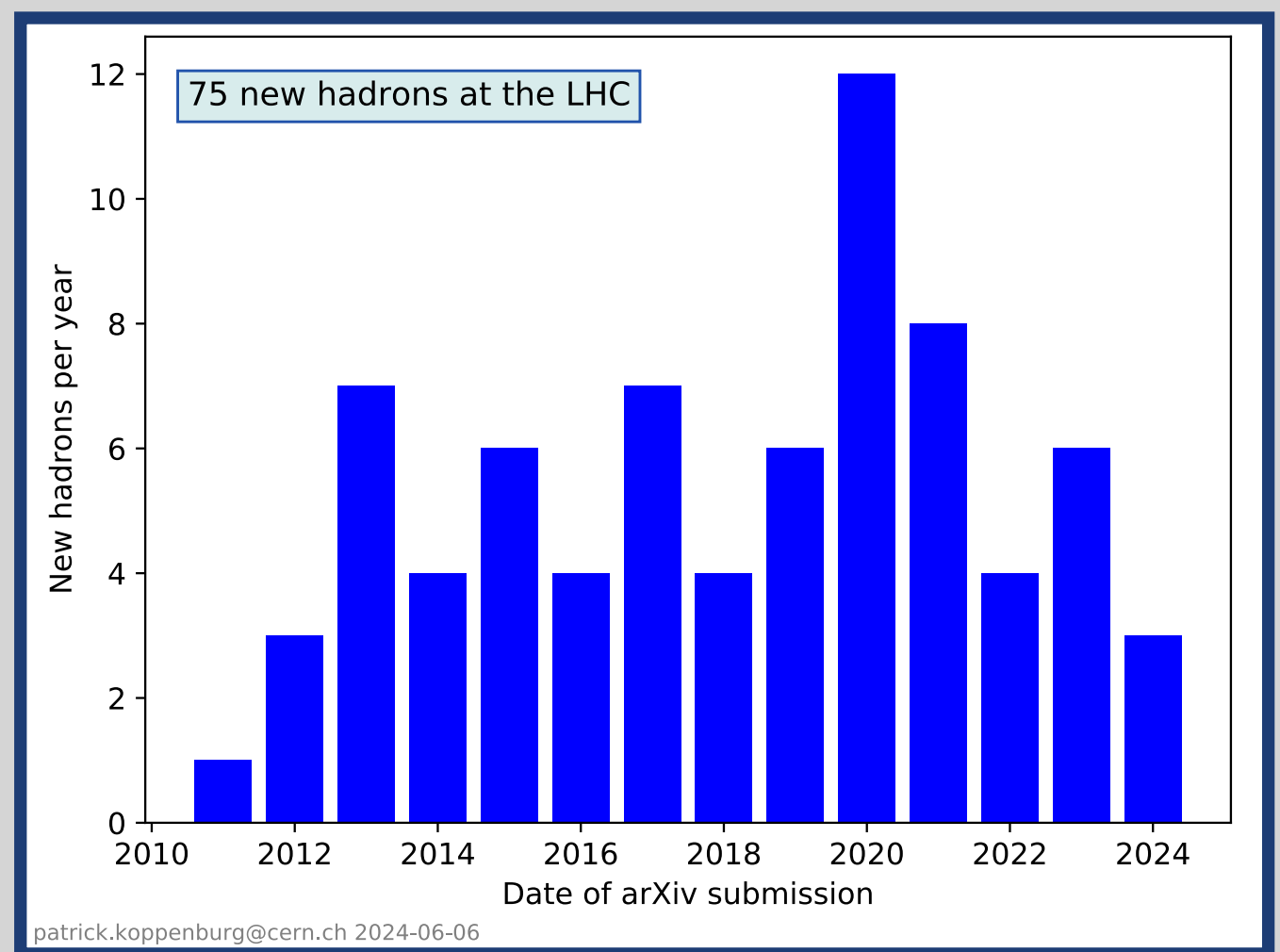


s-channel resonance

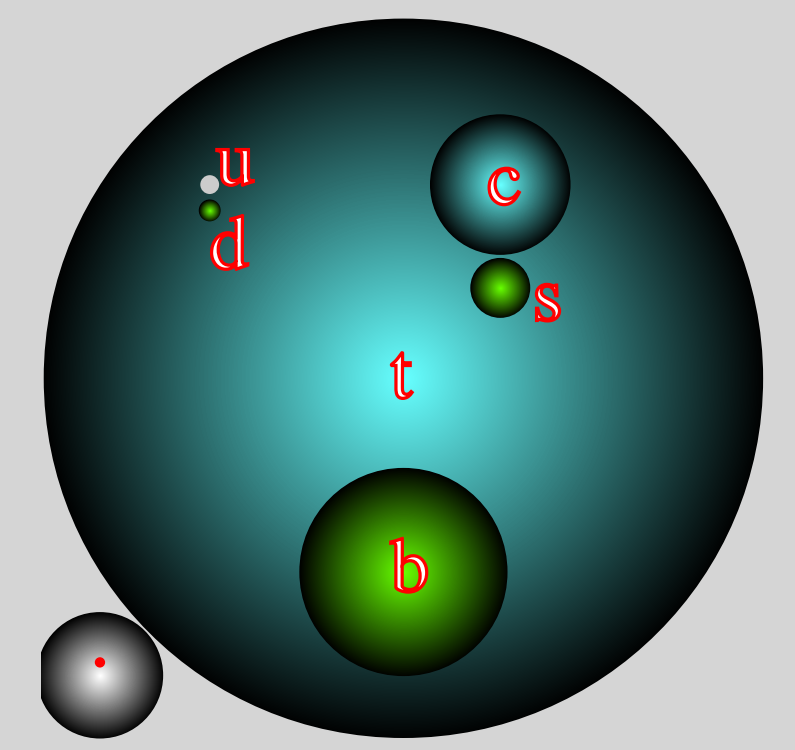
Pole in  ${}^3D_3 - {}^3G_3$



# What's up at LHC??

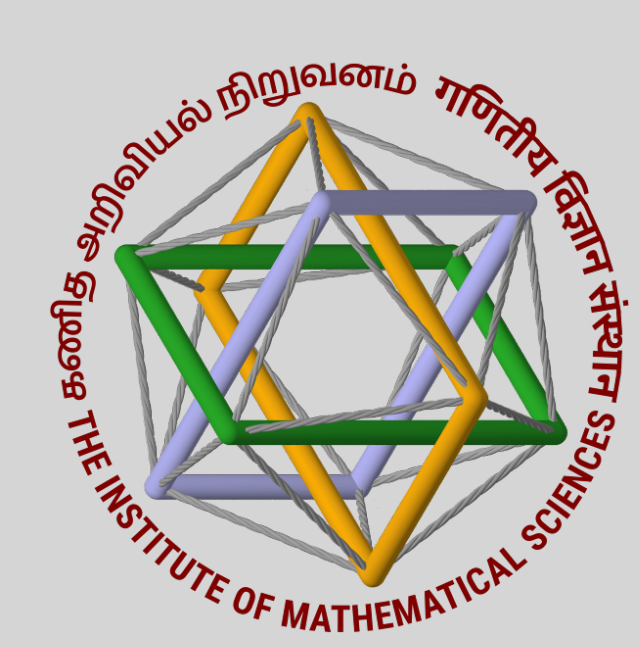


• More interest around heavier hadrons



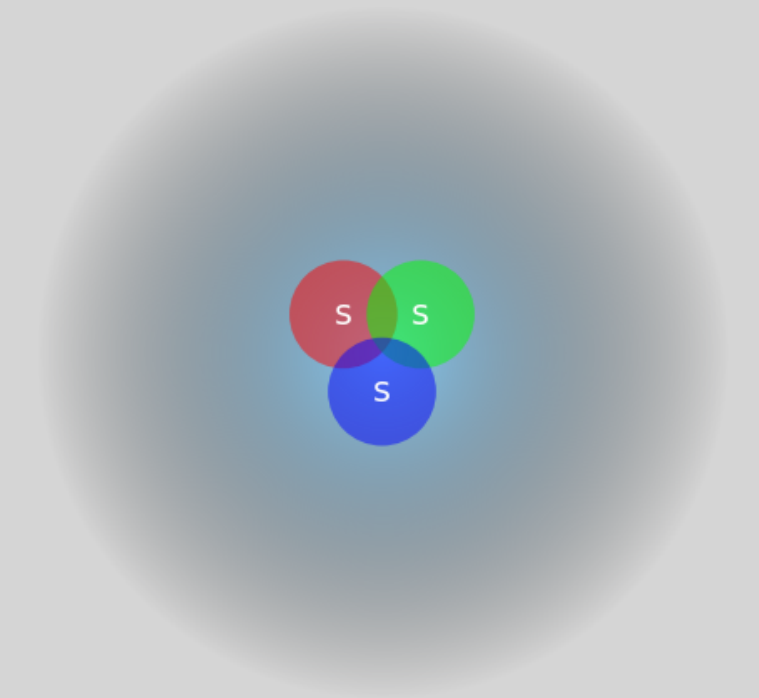
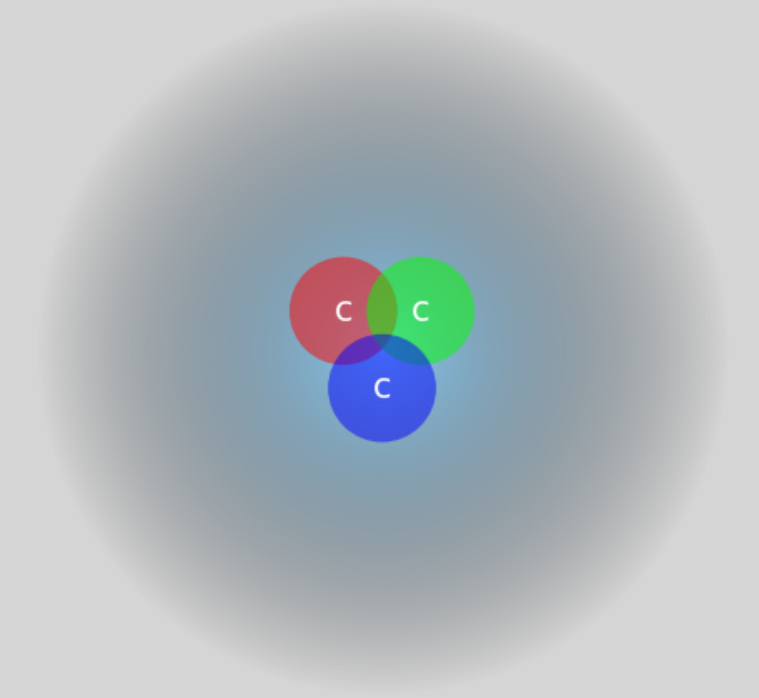
Picture from wiki

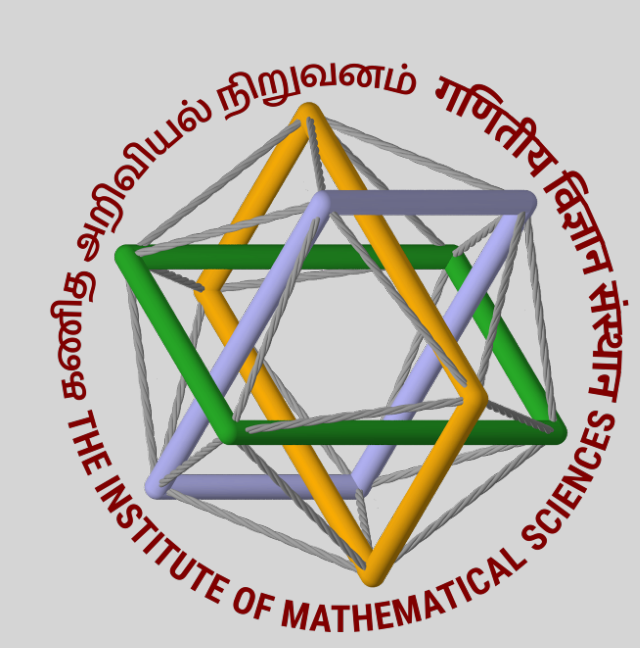
patrick.koppenburg@cern.ch 2024-06-06



# Today's discussion

- Baryons consisting of either strange or charm
- Dibaryons consisting of either strange or charm

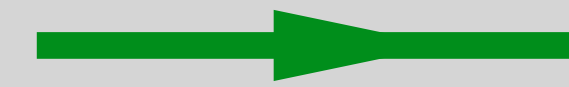




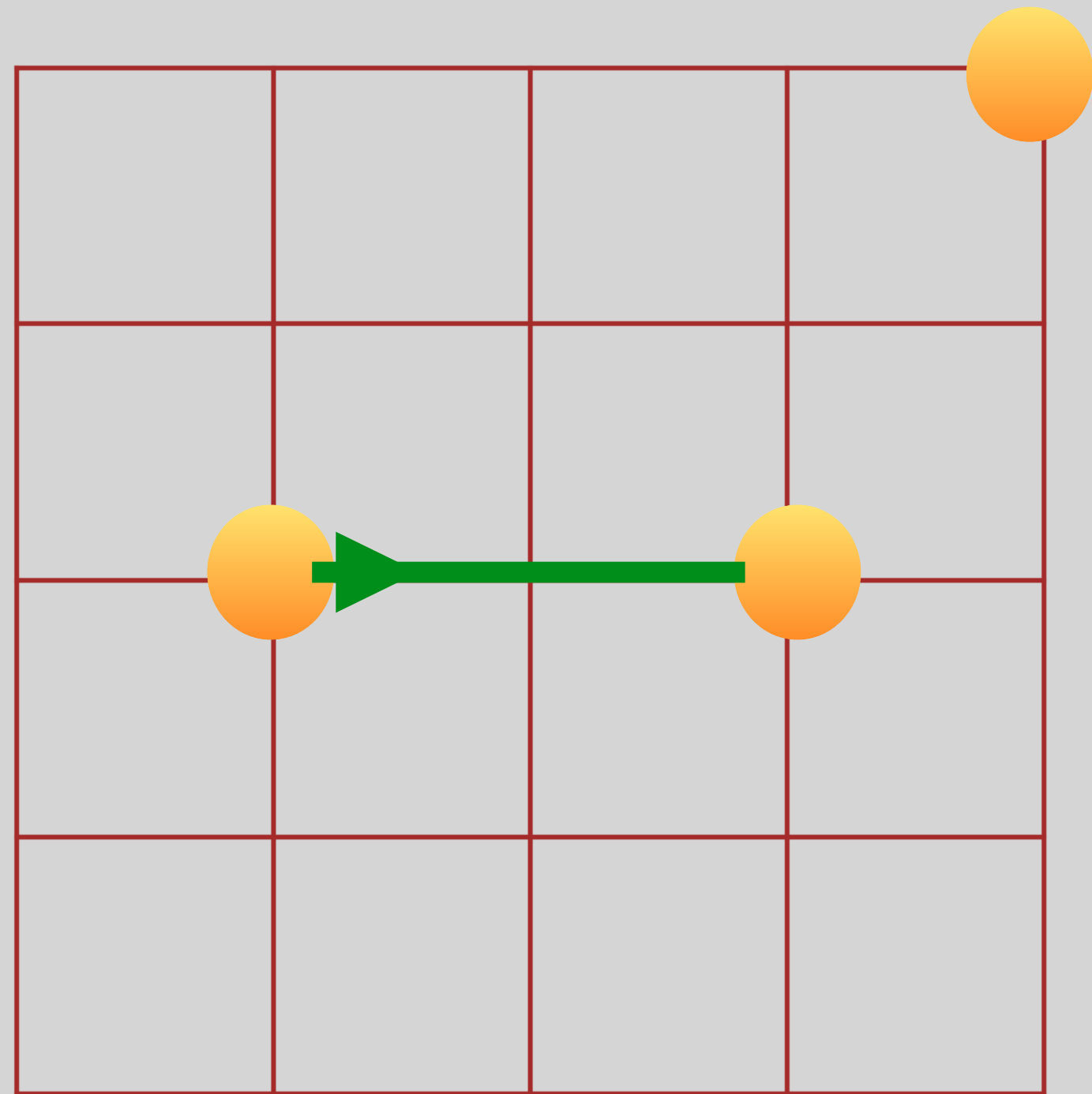
# Lattice QCD



Quark

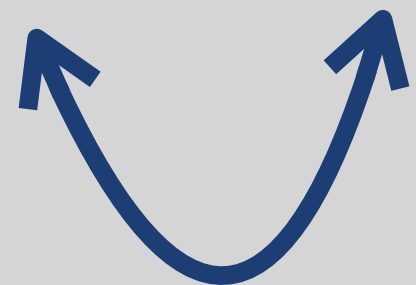


Gluon



$$L = aN_s$$

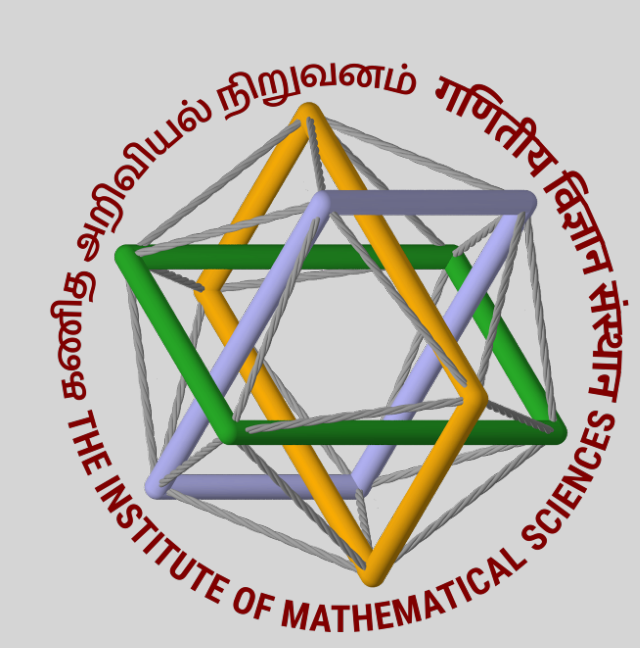
$N_s$  is number of lattice sites



Lattice spacing

- First principles calculation
- Use MCMC to create ensembles of QCD configurations
- Most computational resources spent on generating gauge configurations and evaluating quark propagators



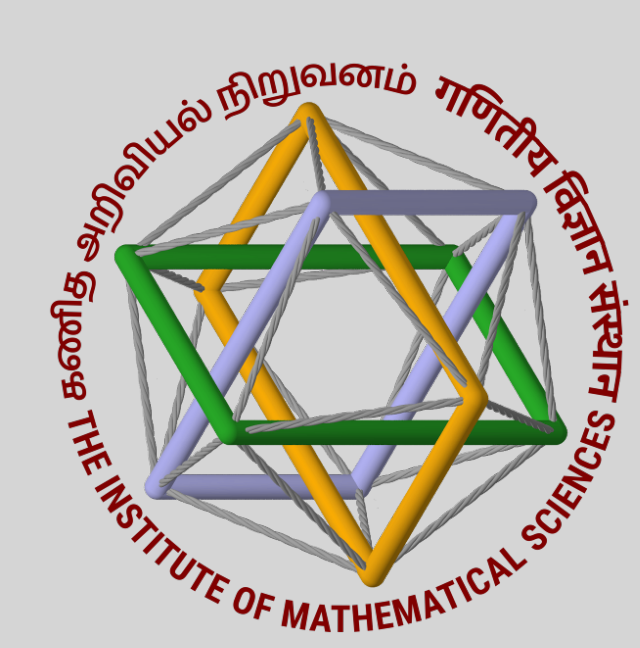


# Hadrons



Announcement  
Talk Series  
August  
By Sasa and Feng-Kun

- \* Desire to understand nature through fundamental forces and interactions.
- \* Baryons and Mesons — Wide range of energy scale
- \* Recent experiments reveal missing baryon states, exotic tetraquark, and pentaquark hadrons.
- \* Lattice hadron spectroscopy predicted numerous bound states, including exotic hadrons.
- \* More progress for bound states stable under strong decay i.e. below threshold or for states closer to threshold



# Deuteron

- Lattice calculations use  $M_\pi \geq 300 \text{ MeV}$

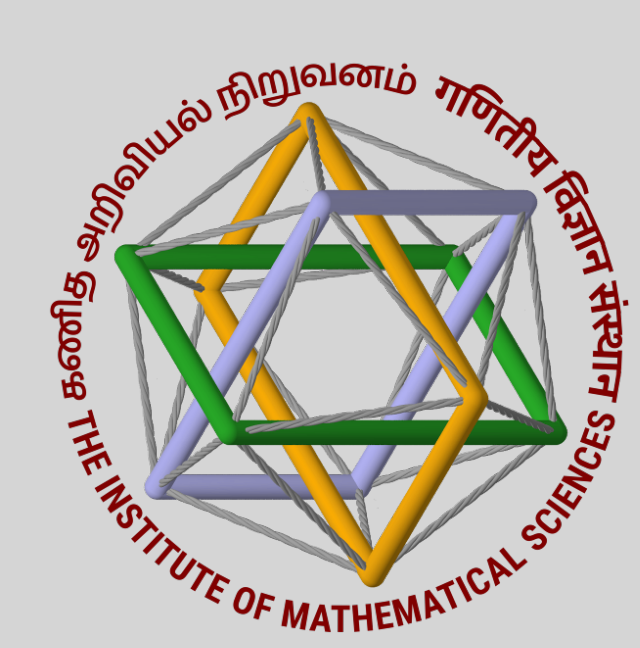
HAL QCD, JHEP 1903 (2019) - Unbound Deuteron, Dineutron  
NPLQCD, PRD 107 (2023) - Bound Deuteron, Dineutron

$$M_\pi \approx 800 \text{ MeV}$$

Why different observations from different Lattice studies?

Will this discrepancy be the case for heavier quarks?

- Discovered around a century back
- Proton (uud) - Neutron (udd) bound state
- Binding Energy = 2.2 MeV



# Why heavier masses ?

$$m_u, m_d \ll m_\pi$$

Light quarks are expensive

$$\text{cost} \propto \left(\frac{1}{m}\right)^{1-2} \left(\frac{1}{a}\right)^{4-6} (L)^{4-5}$$

Lepage, TASI (1989)

Signal ( $m_N$  hadron)

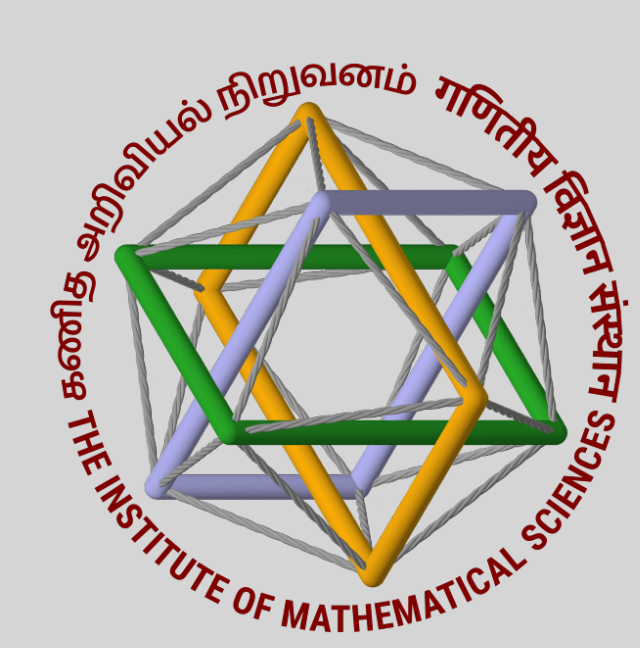
$$\propto e^{-m_N t}$$

Noise

$$\propto e^{-\frac{3}{2}m_\pi t}$$

Error in propagator correlation function dominated by pions because of virtue of lower energy states

Signal to Noise ratio exponentially degrades for  $m_q \rightarrow 0$

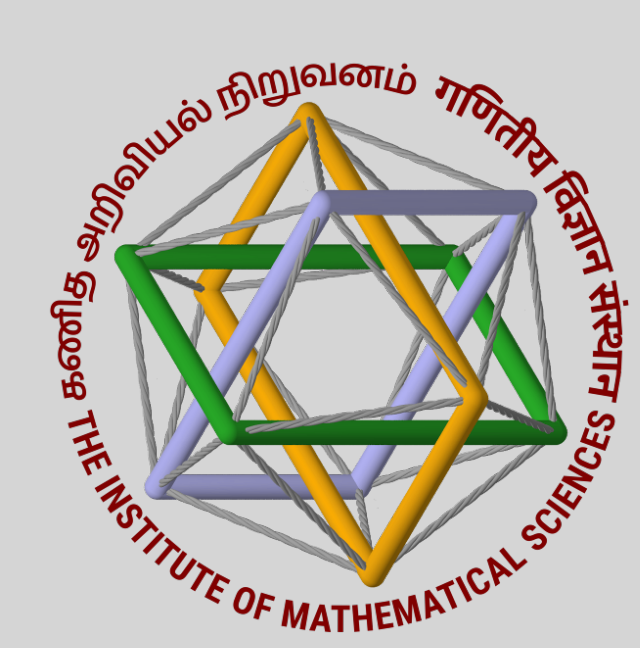


# Lattice - More Requirements

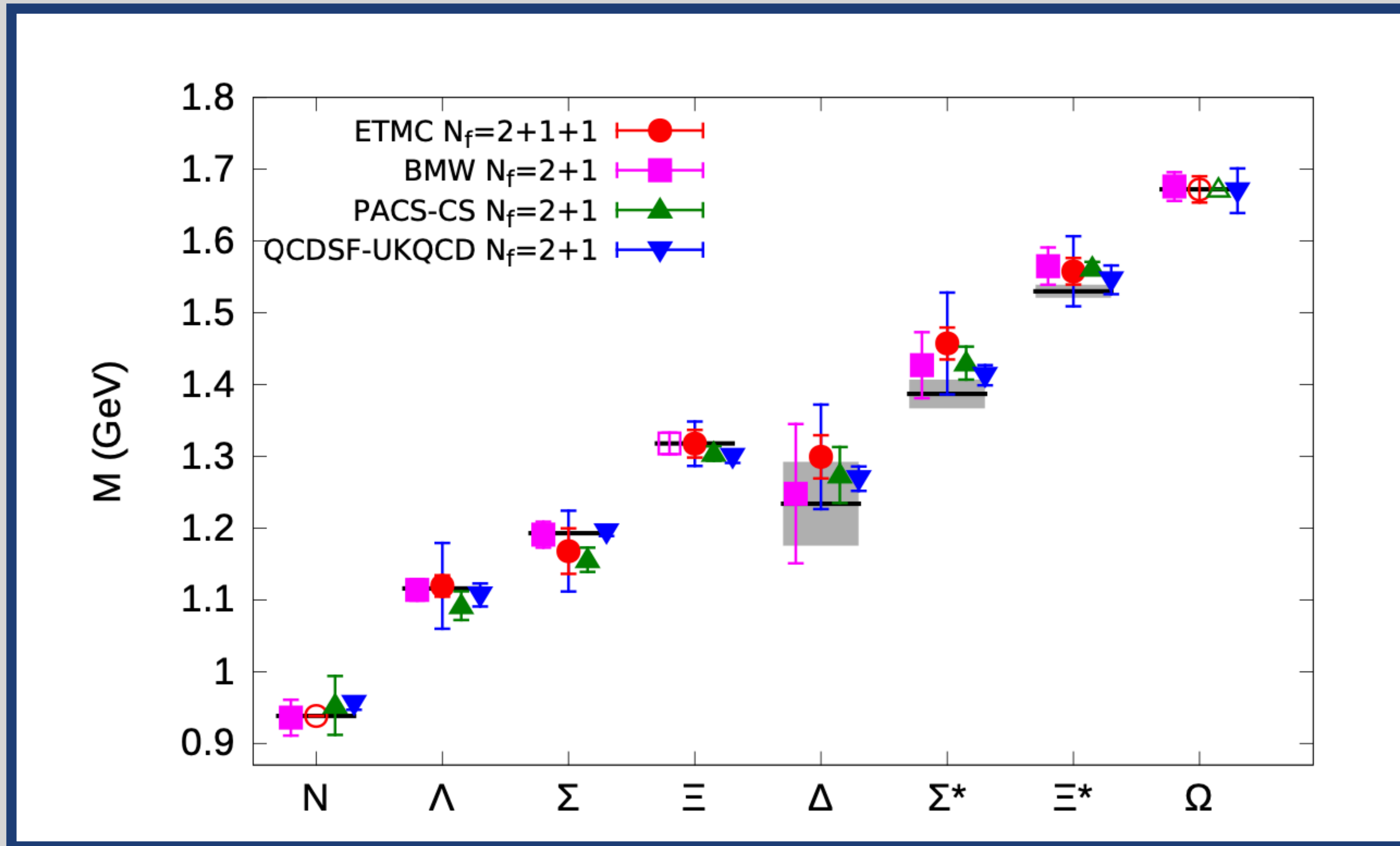
Apart from unphysical heavy light quark masses we need:

- \* Multiple ensembles for continuum limit.
- \* Even more computing power in contractions for exotic hadrons.
- \*  $m_\pi L \geq 4$ , to constraint finite volume effects.
- \* Finer lattices for lesser discretisation errors.

Interplay between choice of  $a$ ,  $L$ ,  $m_\pi$  to have better results

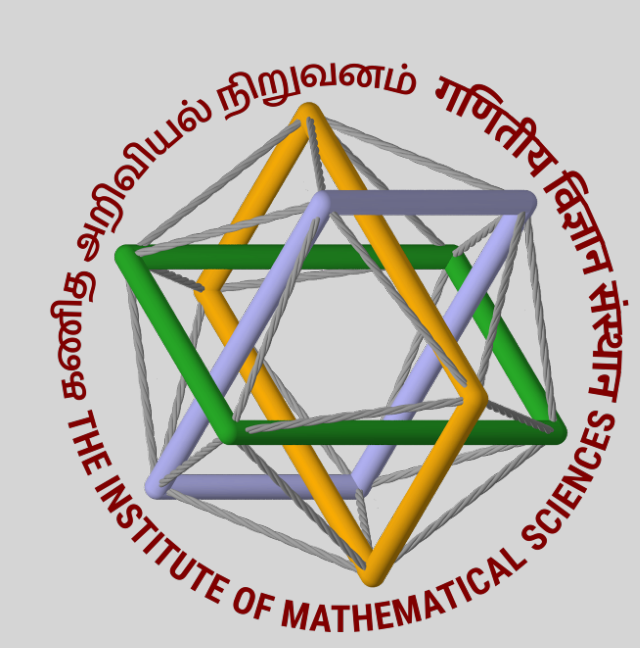


# Baryons from Lattice



- \* Masses of low lying baryons using LQCD.
- \* Results consistent with experiments.
- \* Predictions for hadrons not experimentally measured at that time.
- \* This calculation with  $N_f = 2 + 1 + 1$

Alexandrou, Drach, Jansen, Kallidonis, Koutsou  
PRD 90, 074501 (2014)



# Heavier Baryons from Lattice

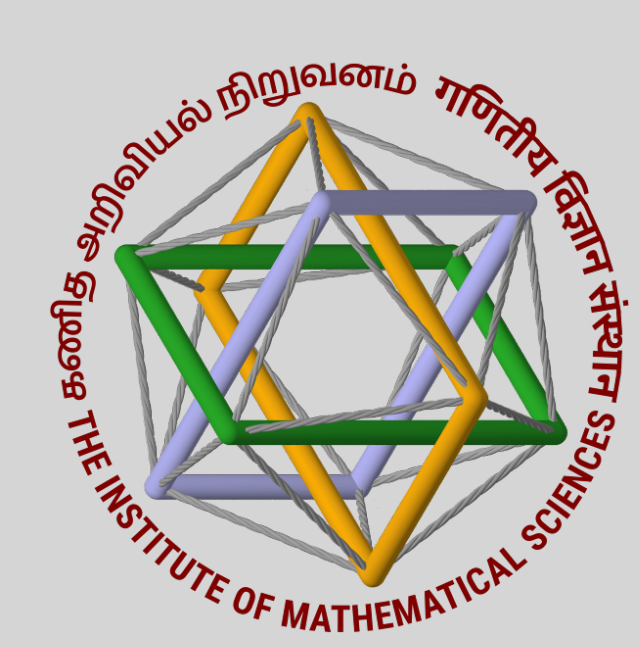
Observations of heavier baryons improved in Lattice calculations over the years (and experimentally):

- \* Bigger (and finer) lattice calculations. In our current calculations we have used  $64^3 \times 192$  lattice with lattice spacing  $a \approx 0.044 \text{ fm}$

- \* Improved algorithms

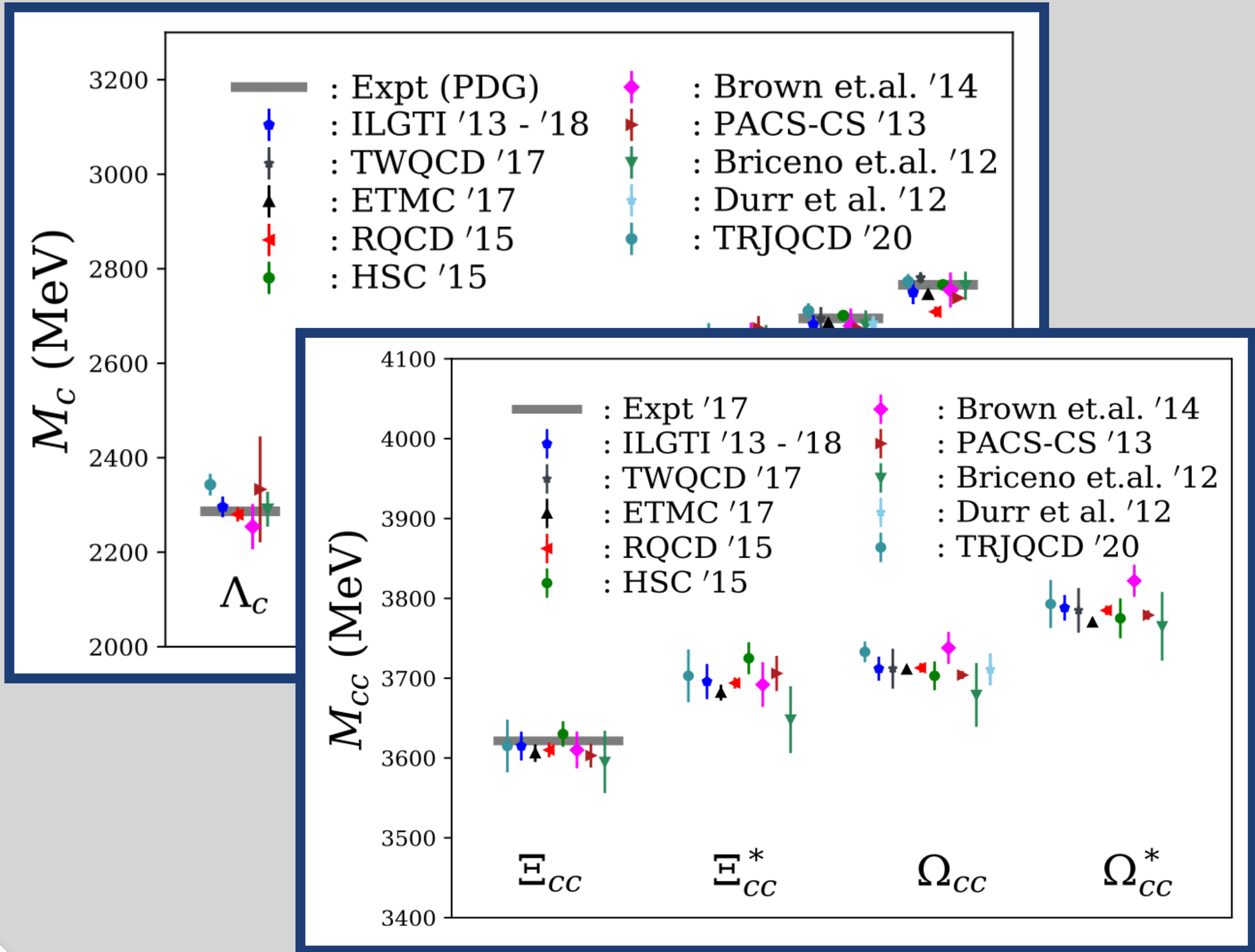
$$N_f = 0 \text{ (quenched)} \rightarrow 2 \text{ (} u, d \text{)} \rightarrow 2 + 1 \text{ (} u, d, s \text{)} \rightarrow 2 + 1 + 1 \text{ (} u, d, s, c \text{)}$$

- \* Growth of machine power from certain GFLOPS to  $\mathcal{O}(10)$  PFLOPS

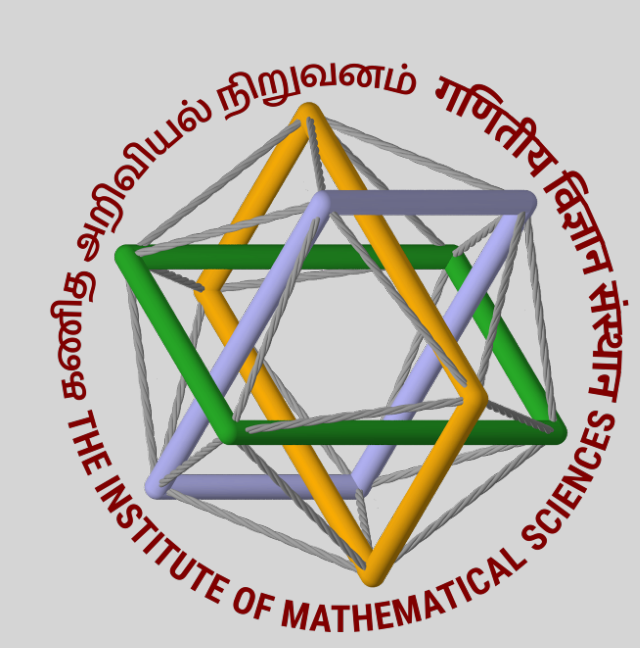


# Heavier Baryons from Lattice

M. Padmanath, CHARM 2020 Talk



- \* Hadrons in charm sector.
- \* Benchmarks for lattice calculations.
- \* Xi double-charm baryon lattice calculation before experimental prediction.
- \* ILGTI calculations matches precisely with other calculations.

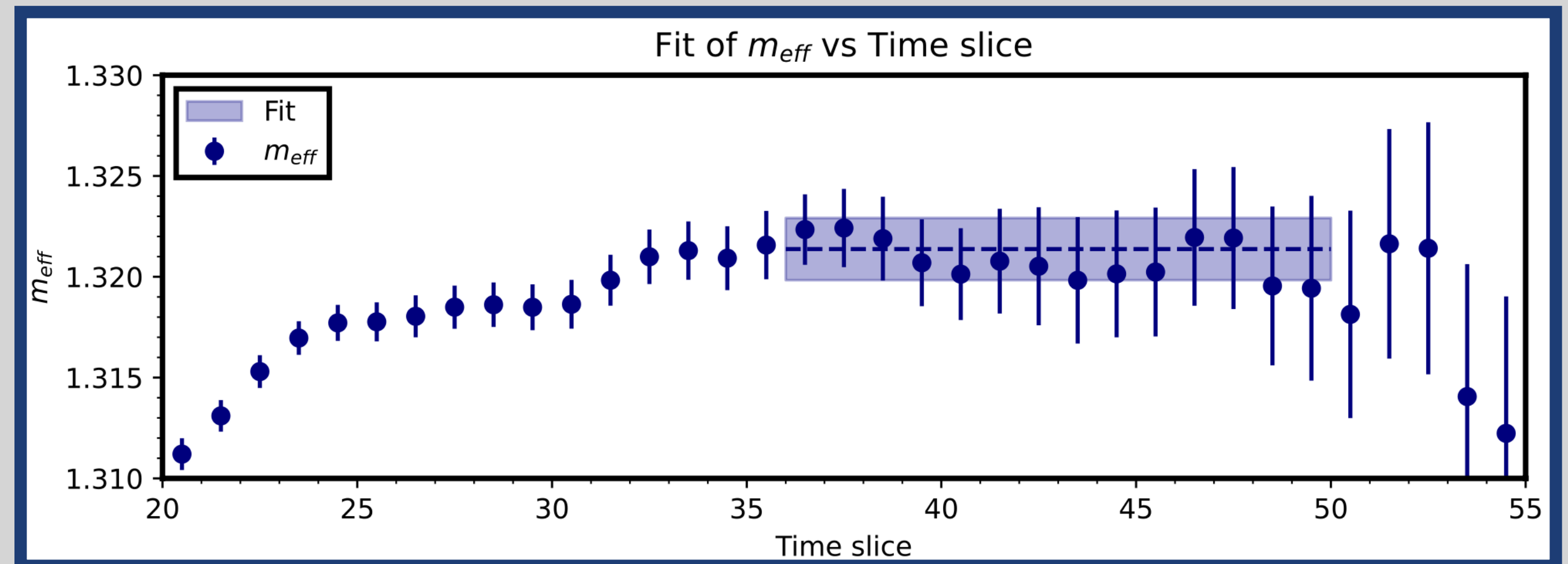


# Masses from Lattice

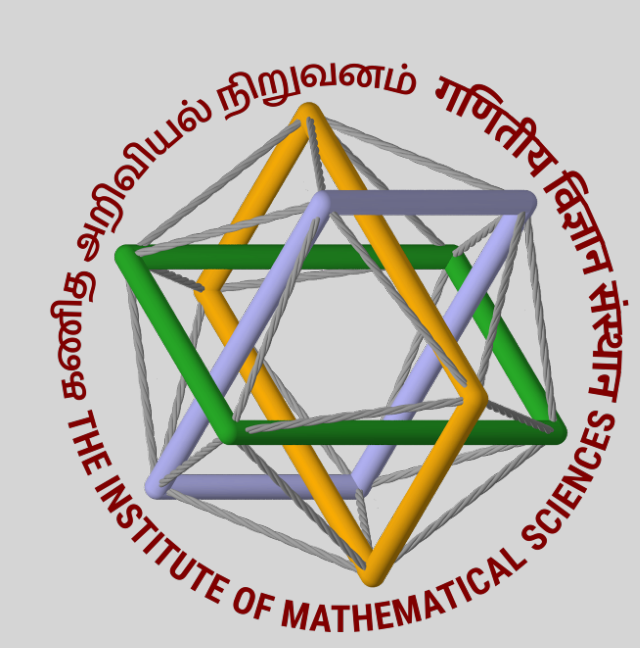
\* Euclidean two point correlator as:  $C_{ji}(t_f - t_i) = \langle 0 | O_j(t_f) \bar{O}_i(t_i) | 0 \rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$

\*  $O_j(t_f)$  and  $\bar{O}_i(t_i)$  are the desired interpolating operators and  $Z_j^n = \langle 0 | O_j | n \rangle$

\* Effective mass =  $\log \left[ \frac{C(t)}{C(t+1)} \right]$







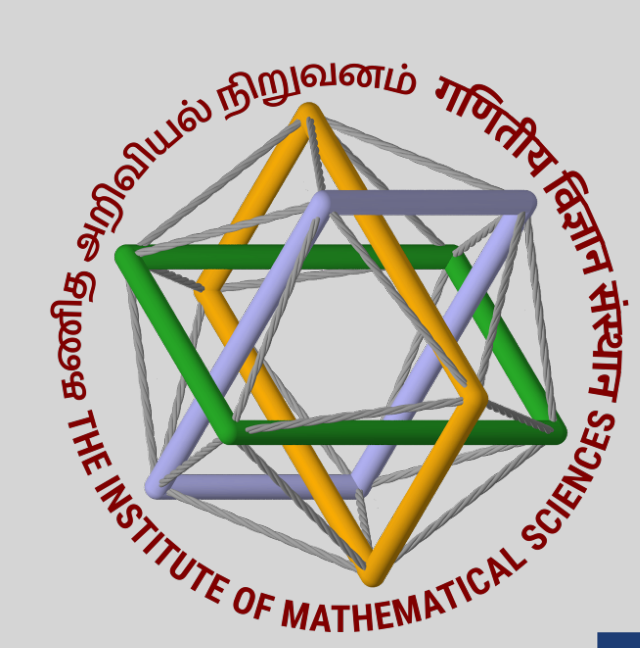
# Operators

Operator for baryon:  $\mathcal{O} = \epsilon_{abc} q_{1,\mu_1}^{a,f_1} q_{2,\mu_2}^{b,f_2} q_{3,\mu_3}^{c,f_3}$

In this work we focus on single flavored baryons

- Total wave function for baryon (fermion) anti symmetric
- Single (symmetric) flavor, color anti symmetric
- Spin must be symmetric -  $3/2$  -  $H^+$  irrep

$H^+$  irrep has two embeddings corresponding to non-relativistic and relativistic operators



# Operators Contraction

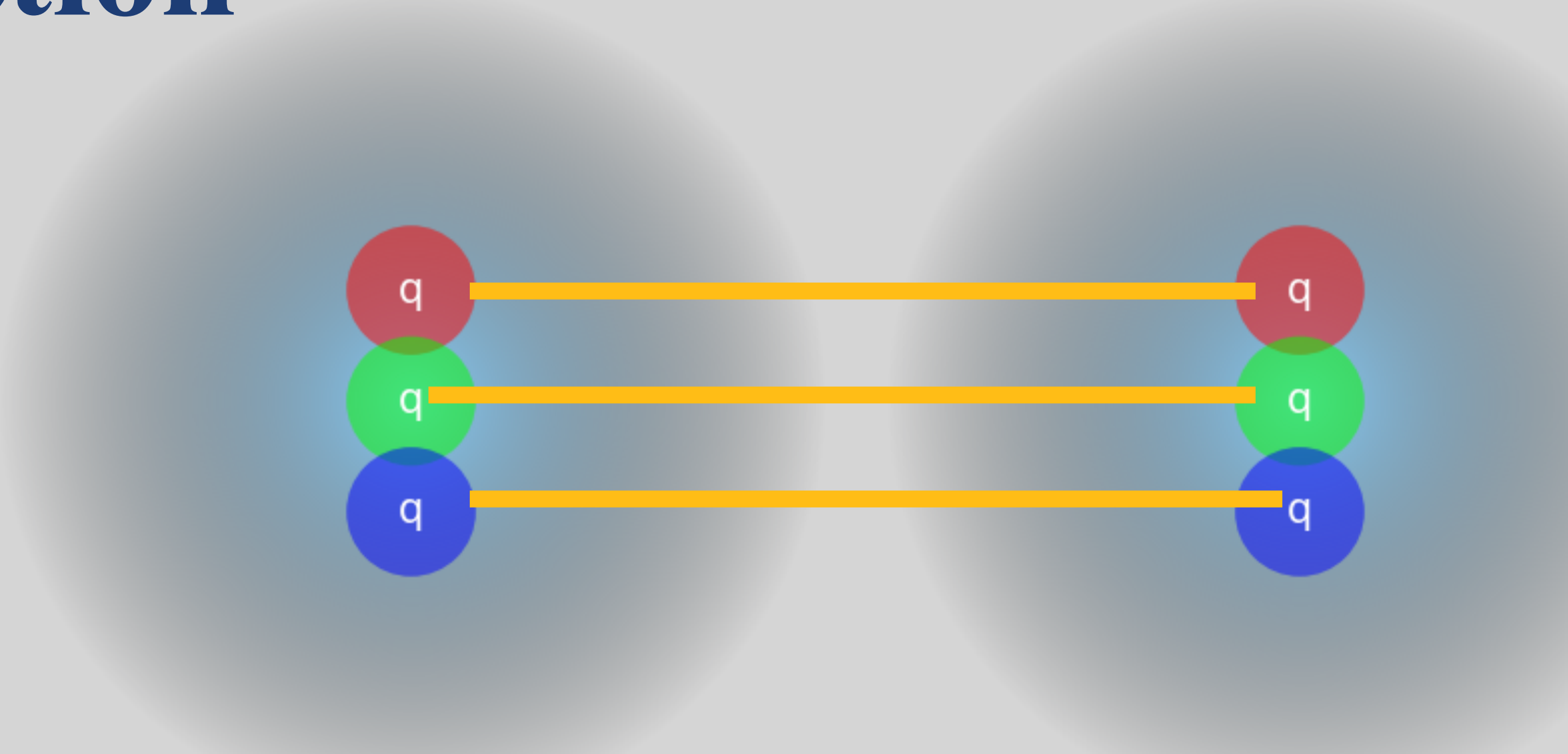
$$\mathcal{O} = \epsilon_{abc} q_{\mu_1}^a q_{\mu_2}^b q_{\mu_3}^c$$

$S_z$	Operator	State
3/2	${}^1H_{3/2}$	111
1/2	${}^1H_{1/2}$	112+121+211
-1/2	${}^1H_{-1/2}$	122+212+221
-3/2	${}^1H_{-3/2}$	222

## Non Relativistic Embedding [N]

$S_z$	Operator	State
3/2	${}^2H_{3/2}$	133+313+331
1/2	${}^2H_{1/2}$	233+323+332+134+341+413+143+431+314
-1/2	${}^2H_{-1/2}$	144+414+441+234+342+423+243+432+324
-3/2	${}^2H_{-3/2}$	244+424+442

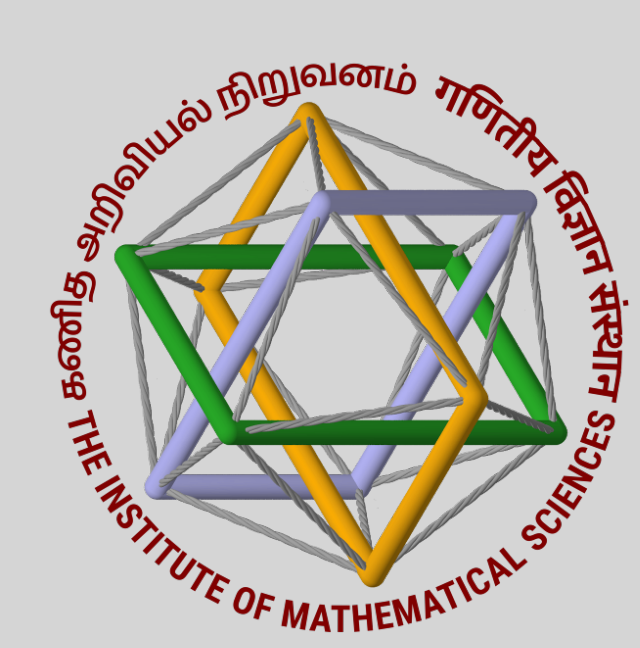
## Relativistic Embedding [R]



### Source slice

### Sink slice

- We can choose different embedding for different operators
- Effectively single contraction



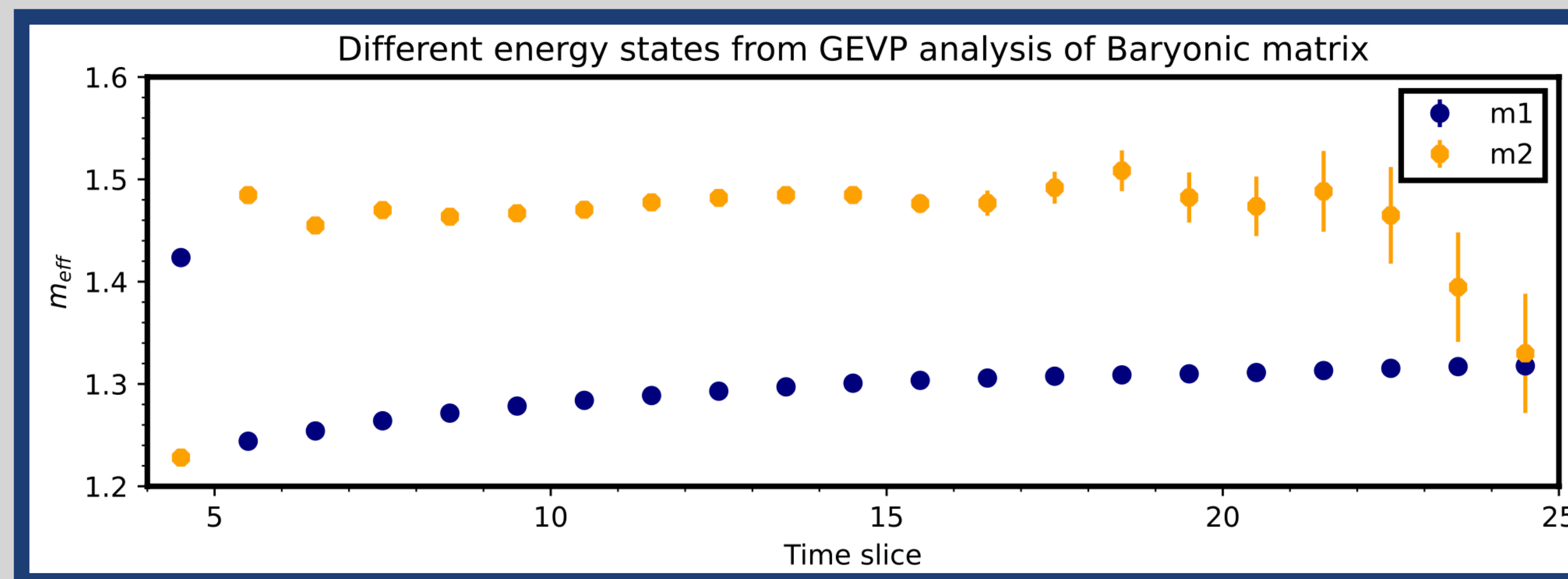
# Operators Contraction

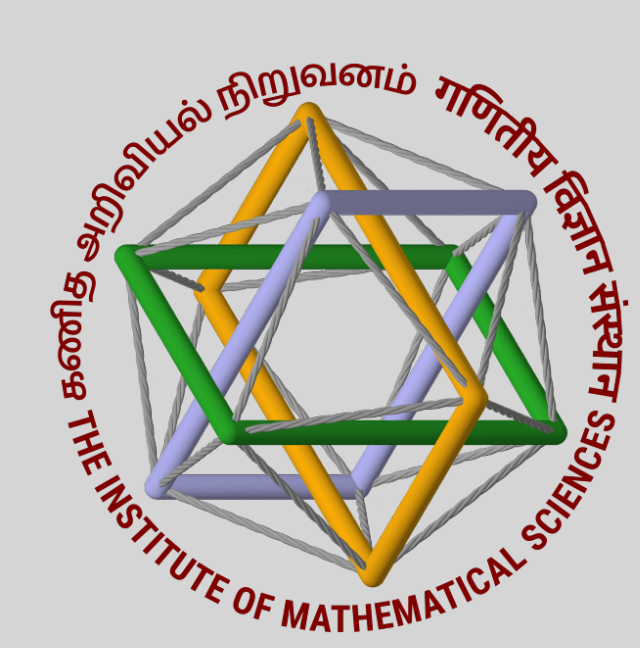
N-N	N-R
R-N	R-R

Baryonic matrix constructed out of both non-relativistic and relativistic operators on which we apply GEVP

- Larger overlap if we use just N-N operator combination
- We use full matrix to show that there is no excited state contamination

- Time reversed backward propagator used to double data
- Negative parity calculation also done

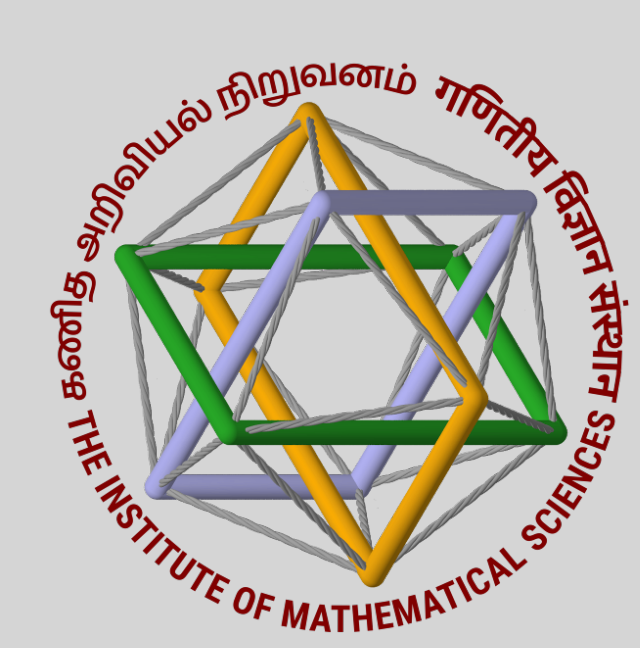




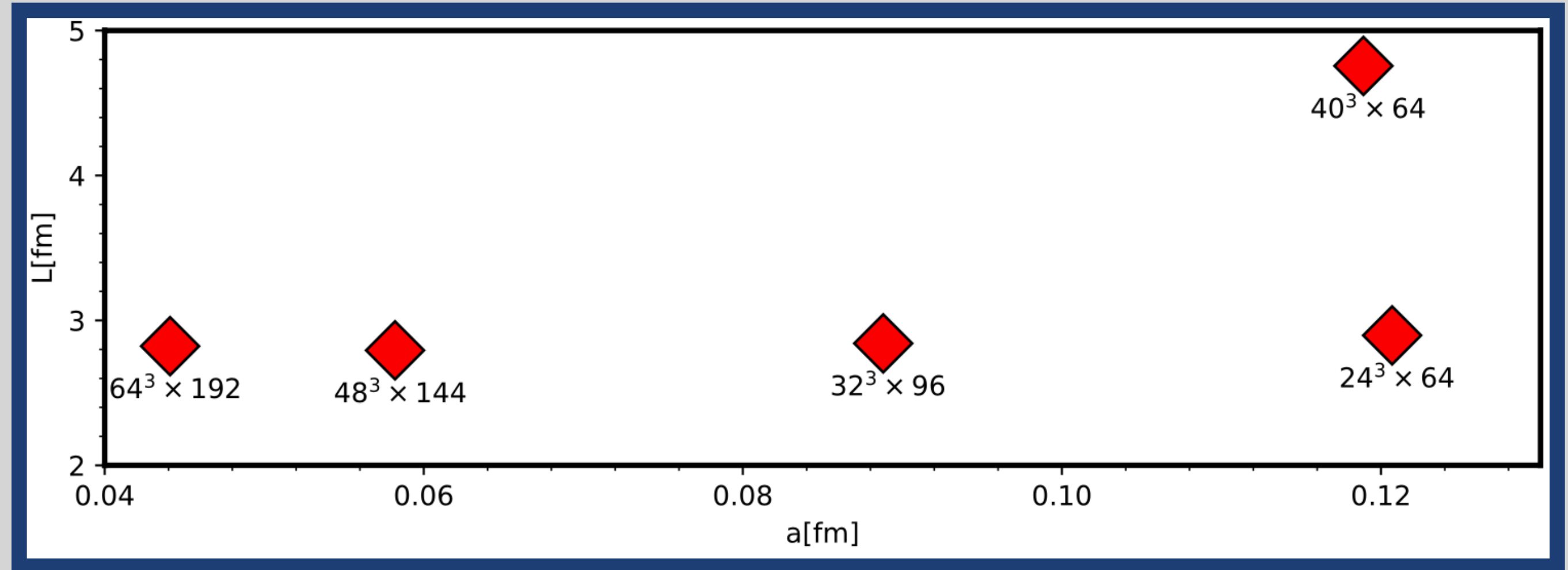
# Lattice Setup

Bazavov et al., PRD 87 (2013) 5, 054505

- Overlap action on background of Highly Improved Staggered Quark (HISQ) gauge configurations.
- Strange and charm masses set at physical values.
- Up and down set as degenerate masses heavier than physical values.
- Finest lattice used  $a \approx 0.044 \text{ fm}$  with Volume as  $64^3 \times 192$



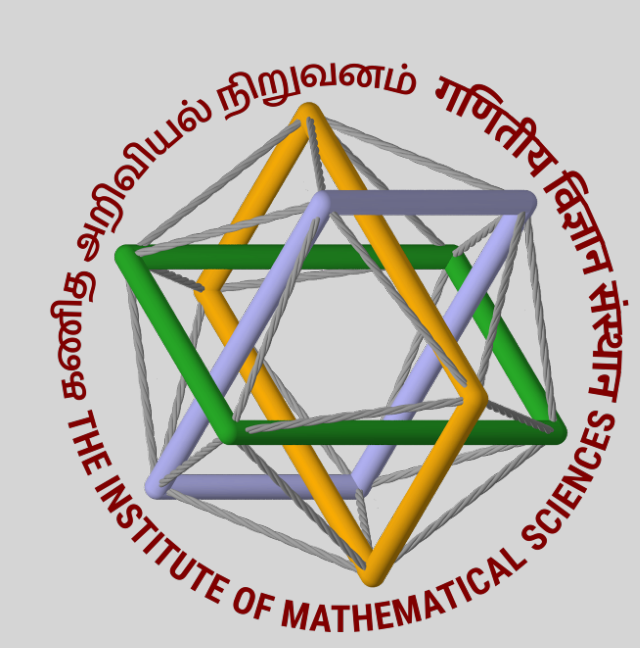
# Ensembles



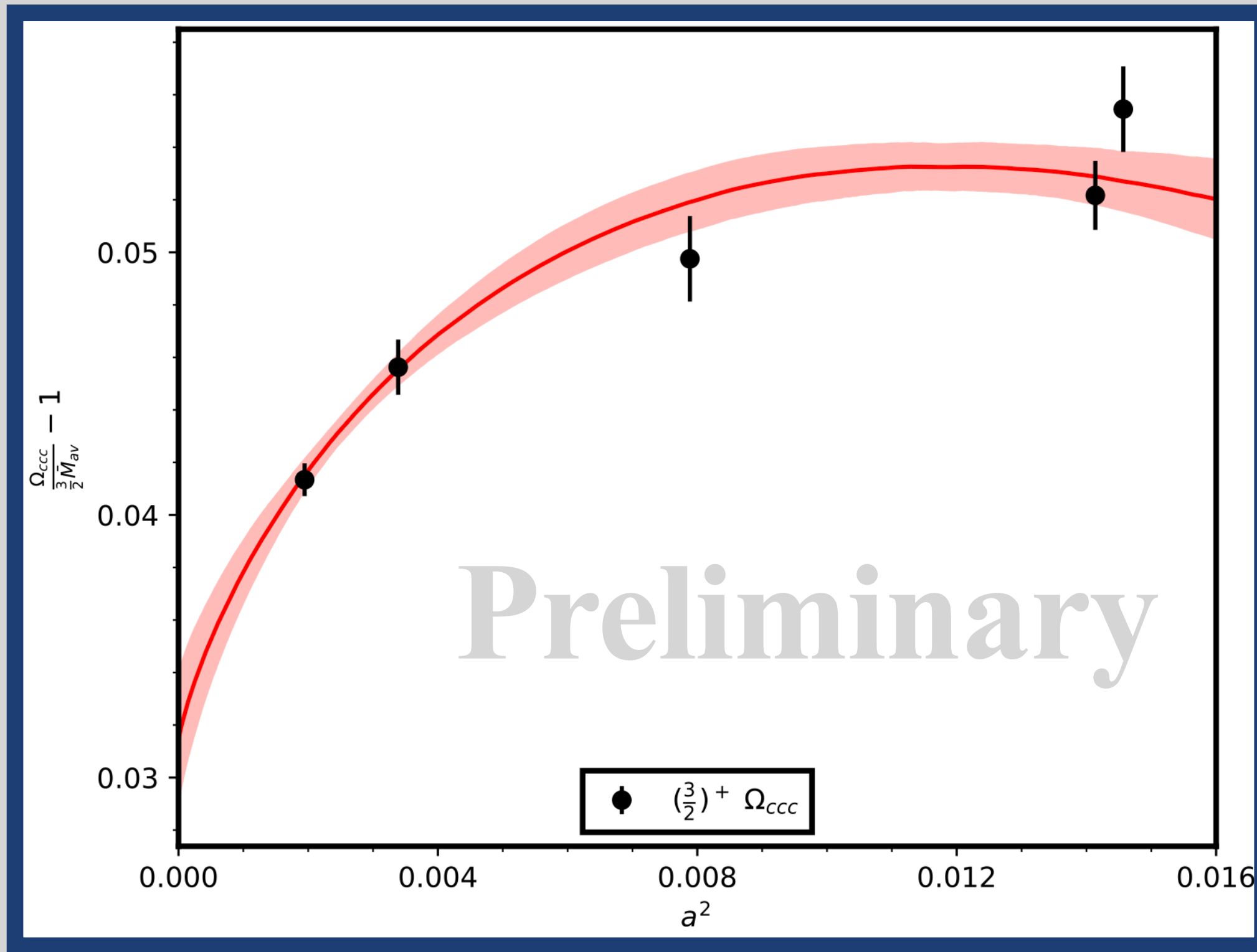
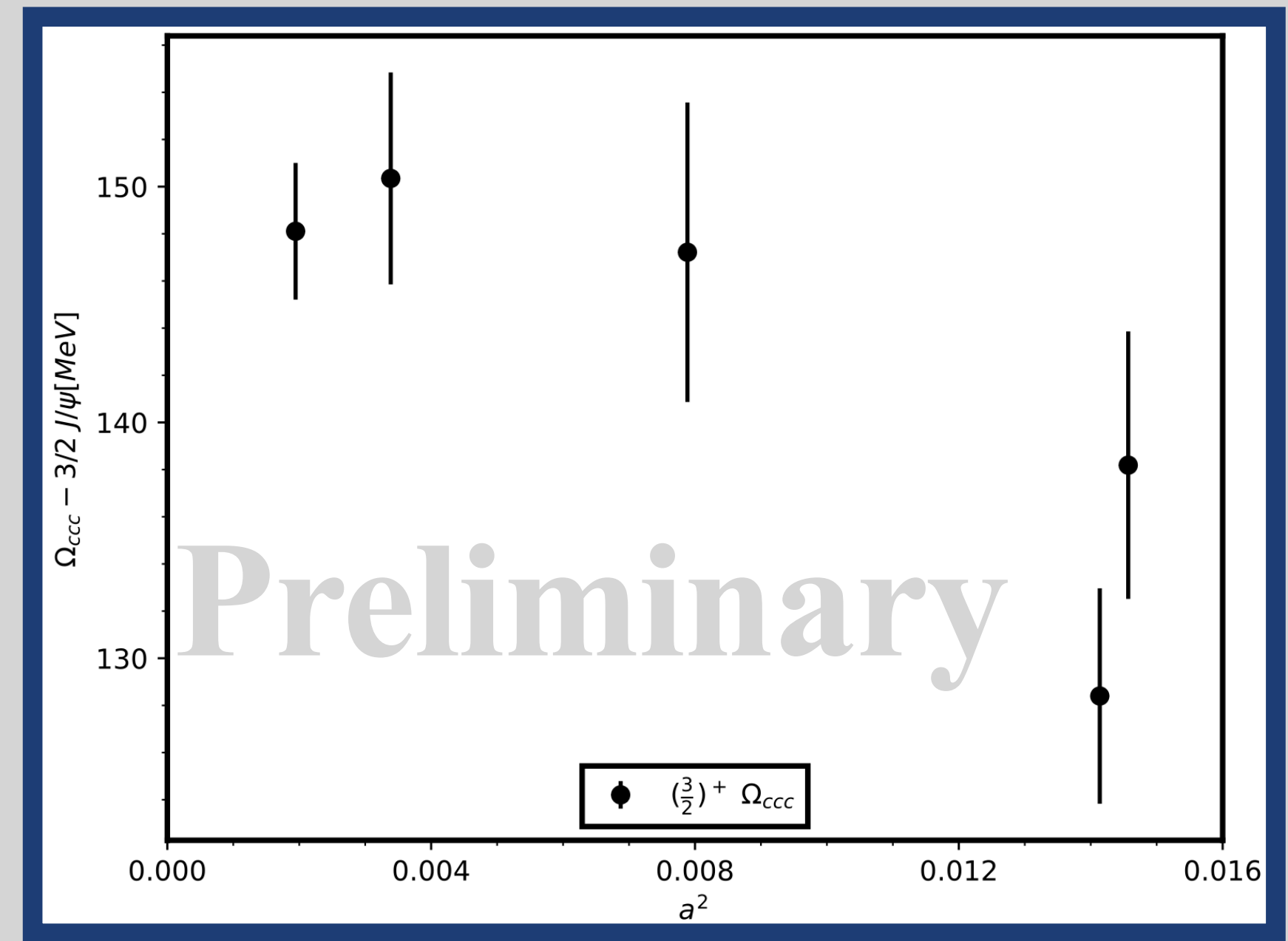
$$\bar{M}_{av} = \frac{1}{4} \left( M_{n_c} + 3M_{J/\psi} \right)$$

$$M_h = \frac{3}{2} \bar{M}_{av}$$

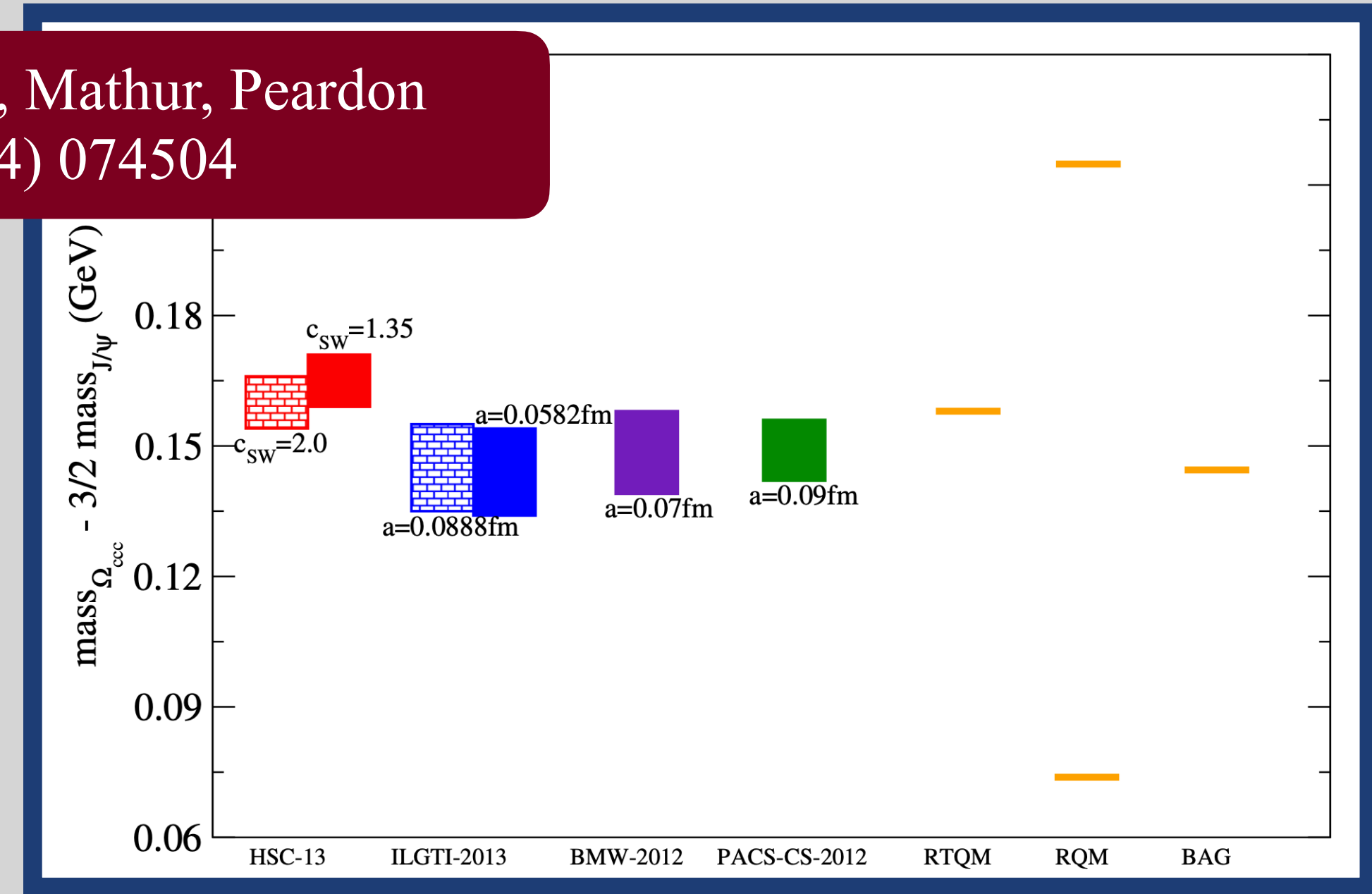
- Configurations generated by MILC.
- Discretisation effects highest in triply charmed baryons.
- Instead of effective mass in charmed baryons, we will plot effective splittings.



# $\Omega_{ccc}$ Baryon



Padmanath, Edwards, Mathur, Peardon  
PRD 90 (2014) 074504

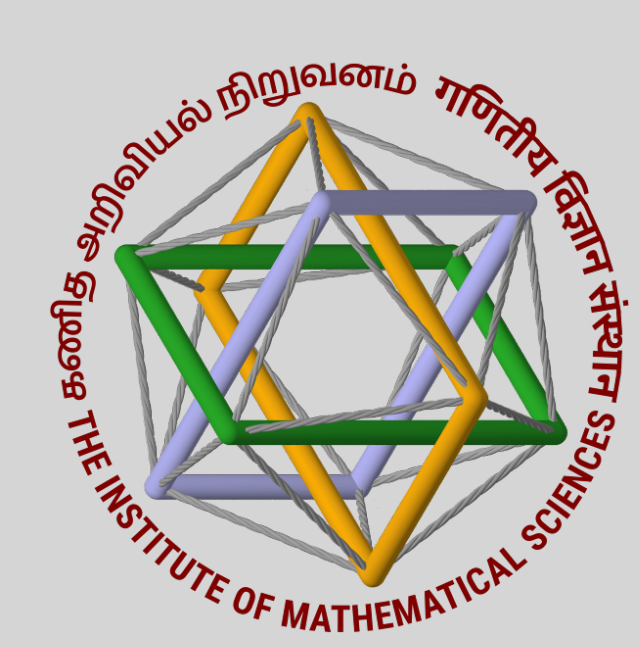


$A + Ba^2 + Ca^2 \ln(a)$  fit

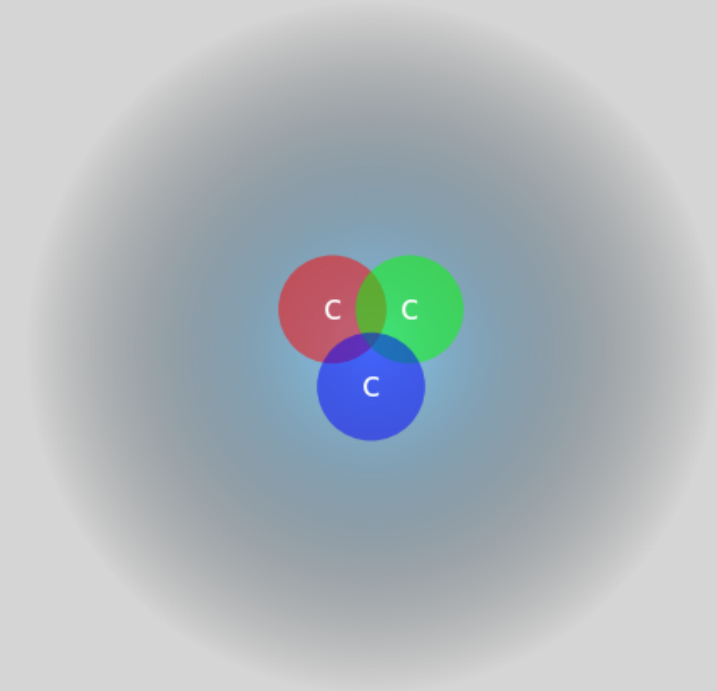
$$\bar{M}_{av} = \frac{1}{4} (M_{\eta_c} + 3M_{J/\psi})$$

In Progress: NSD, Padmanath, Mathur

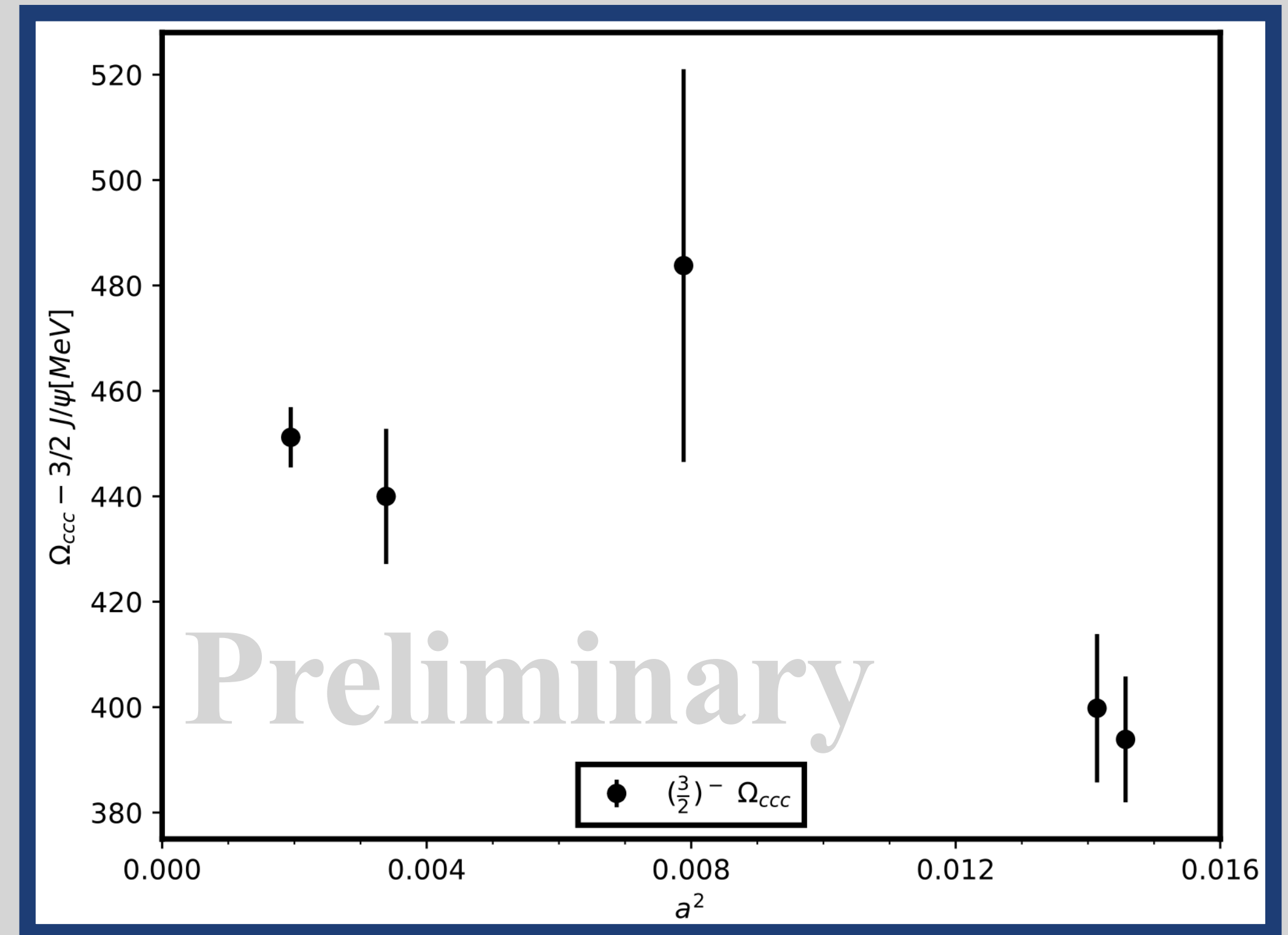
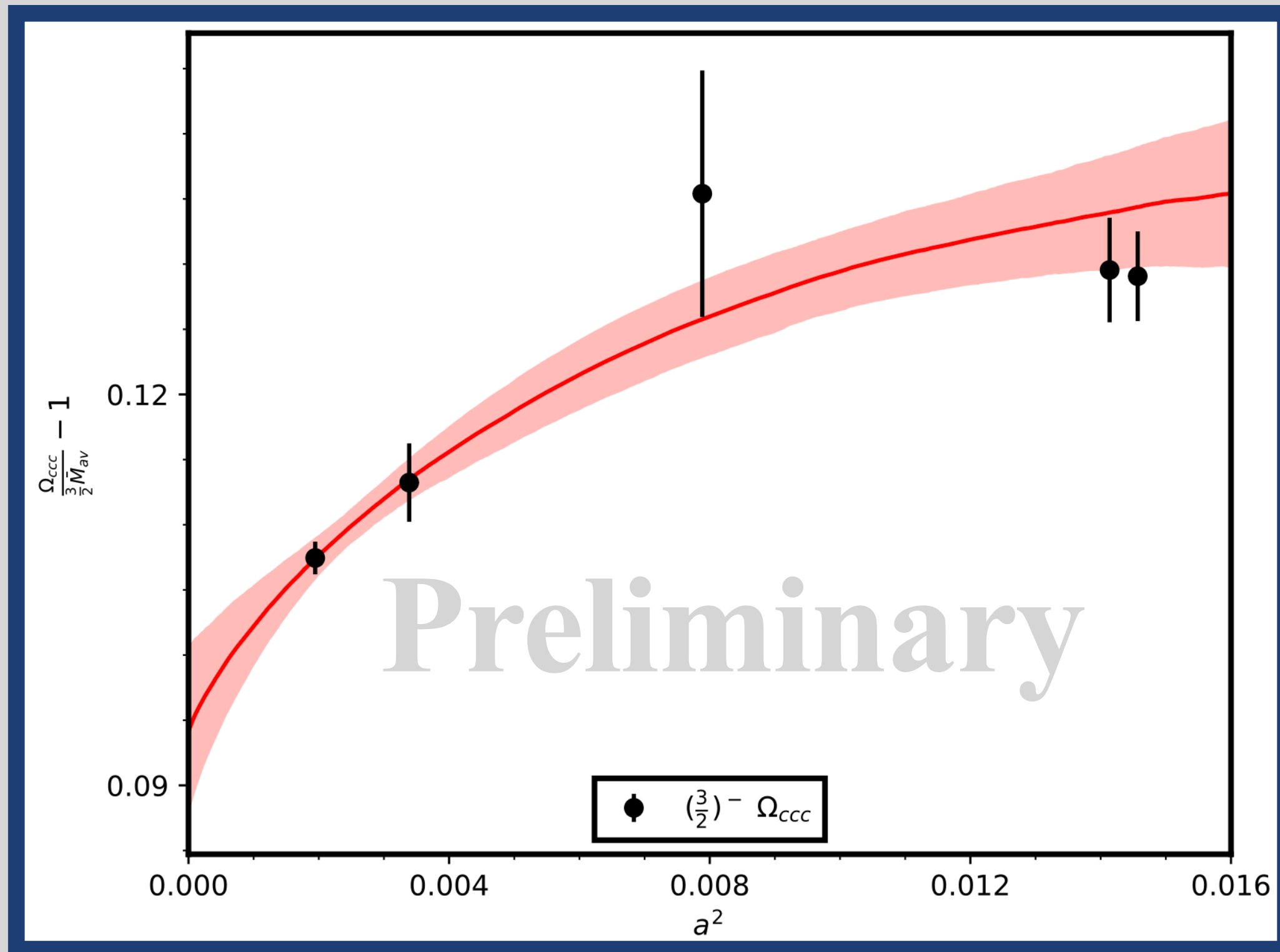
Navdeep Singh Dhindsa 15/07/2024



# $\Omega_{ccc}$ Baryon

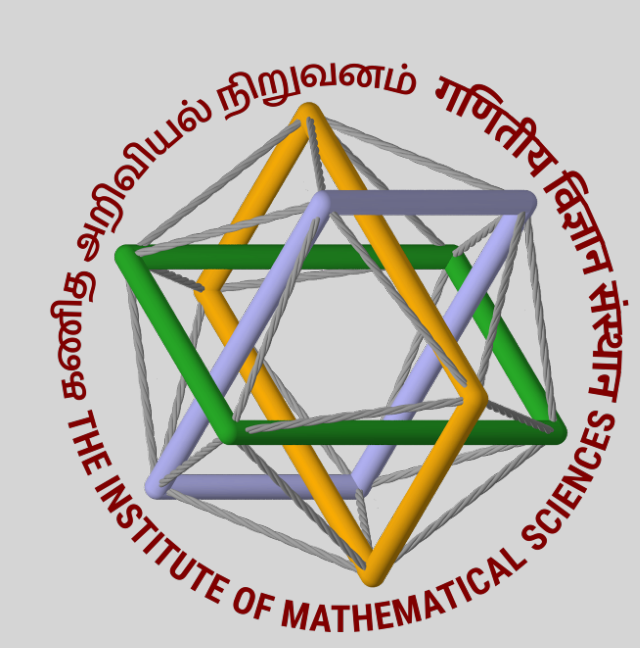


Will we see experimental observation of  $\Omega_{cc}$  and  $\Omega_{ccc}$  soon?

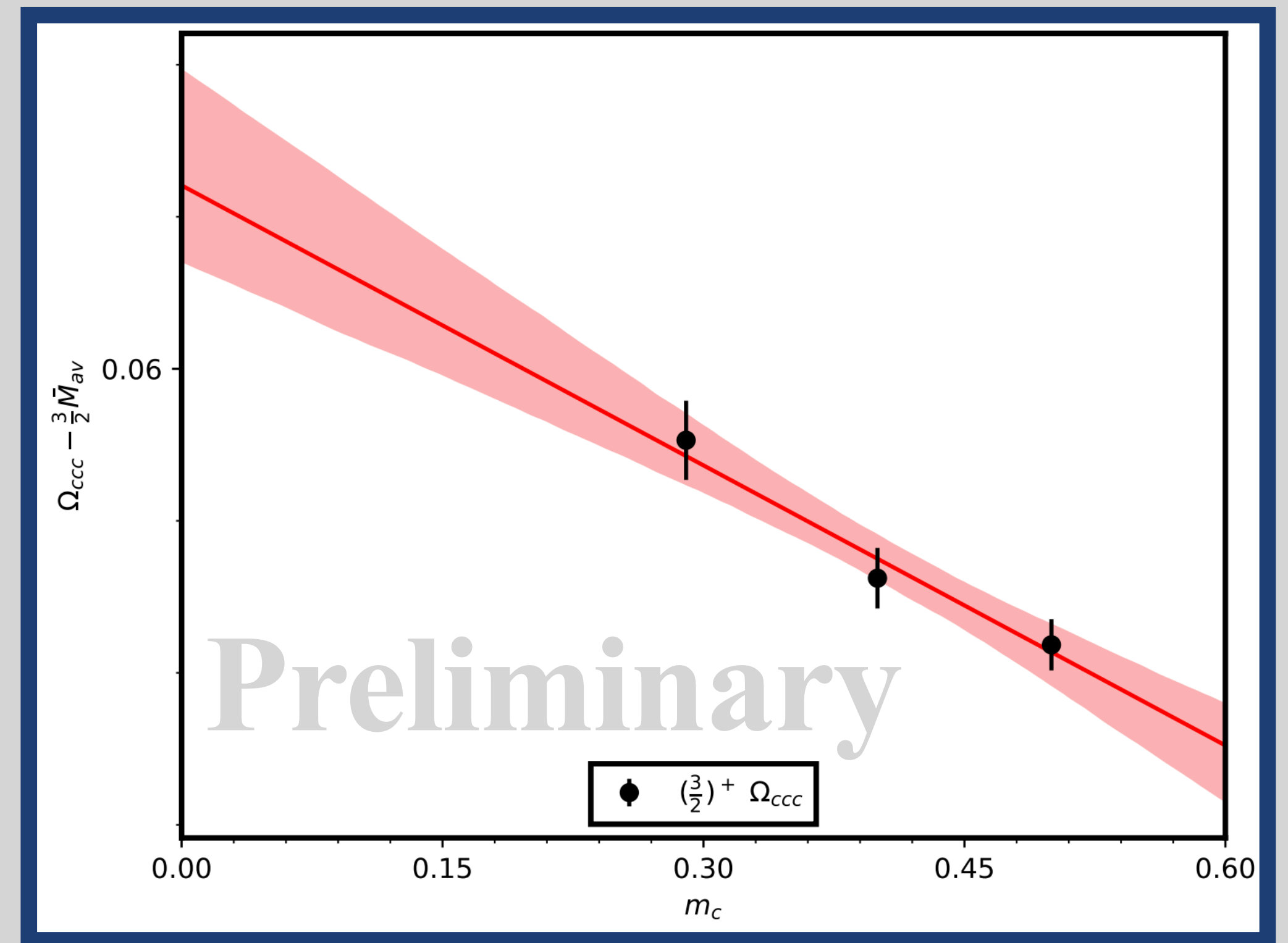
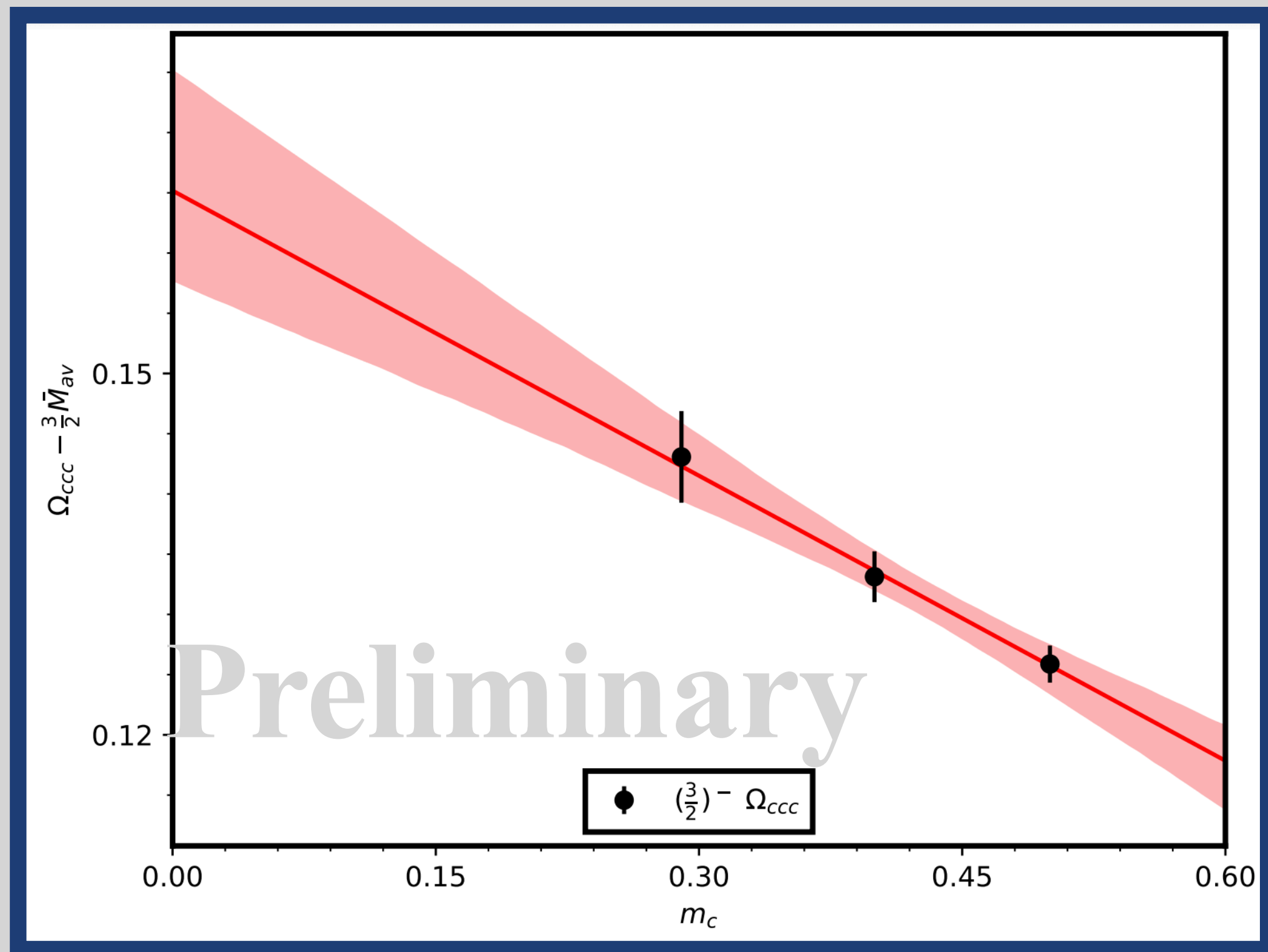
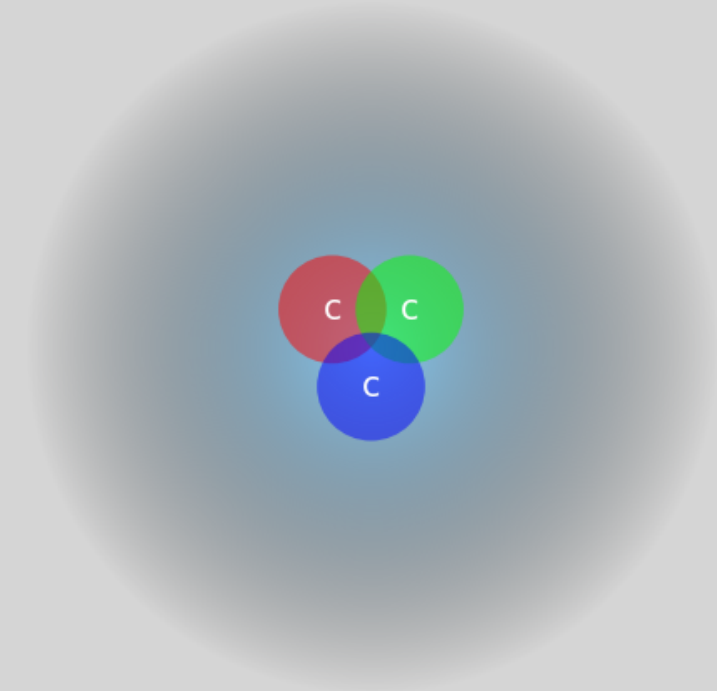


Negative Parity





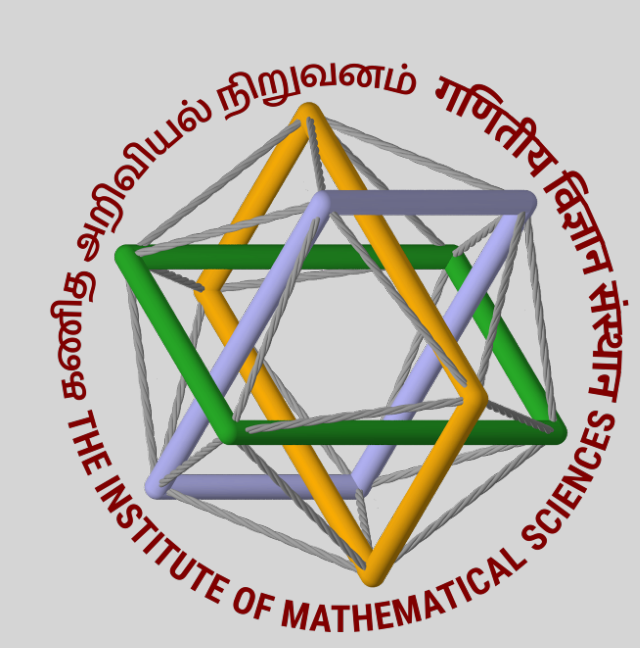
# $\Omega_{ccc}$ Baryon



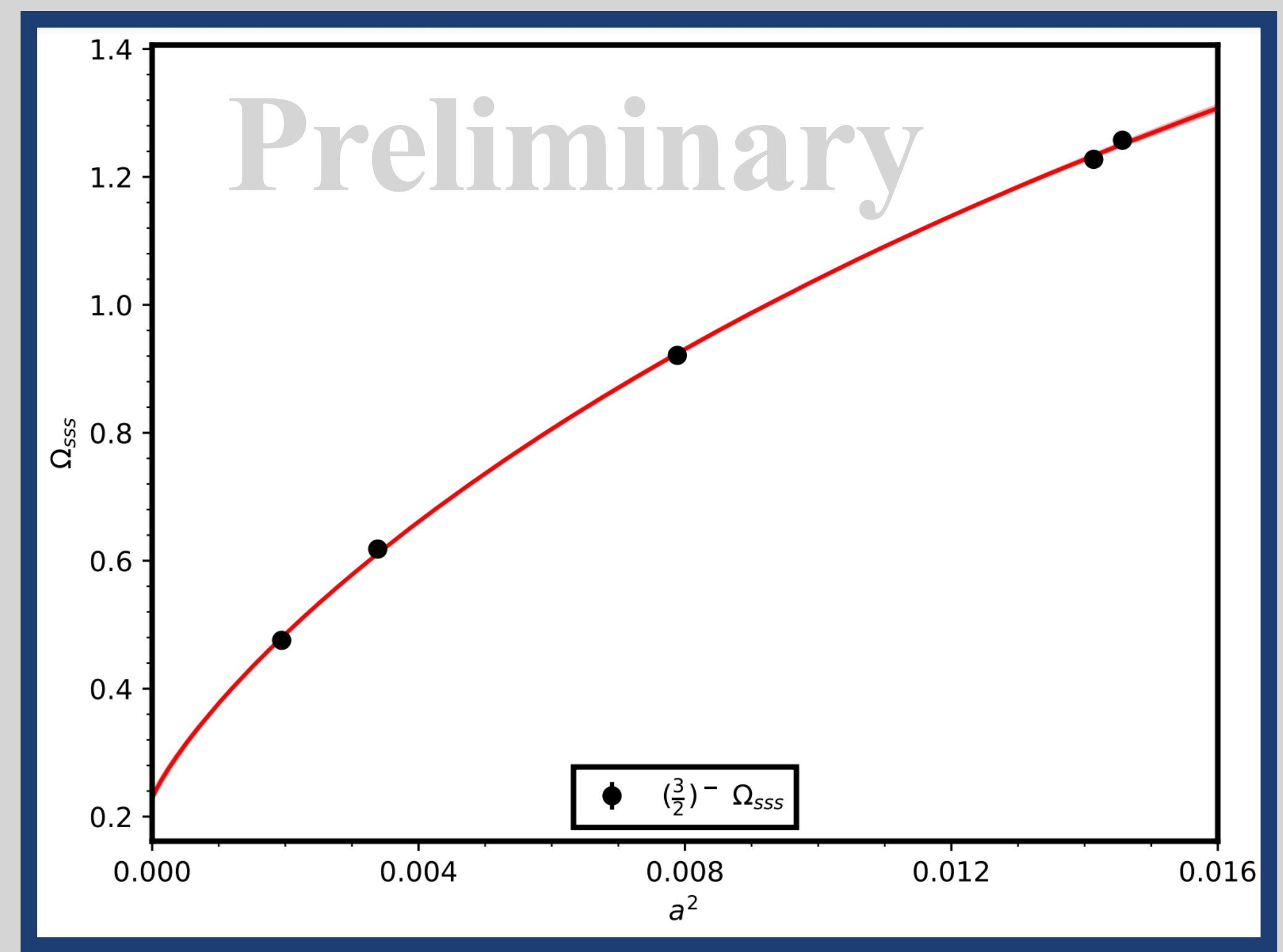
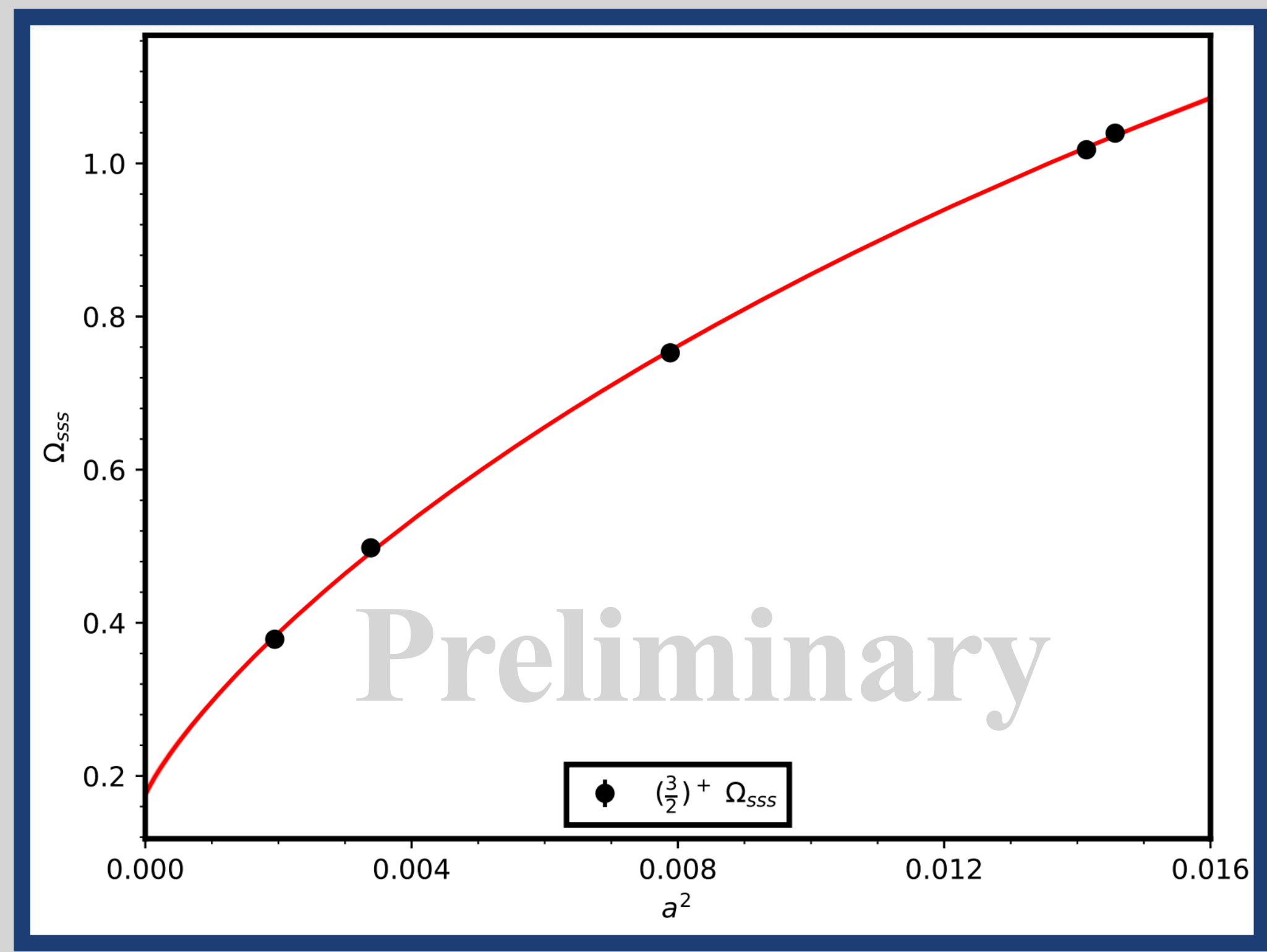
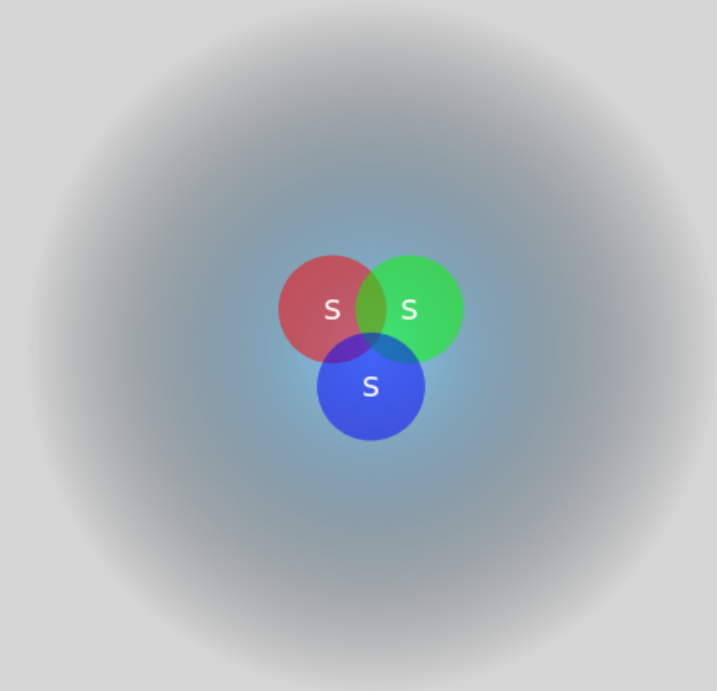
Tuning charm mass

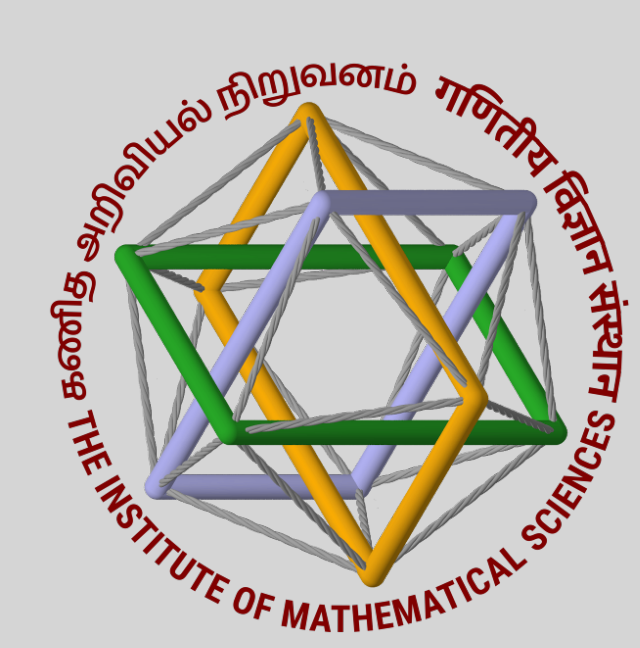






# $\Omega$ Baryon



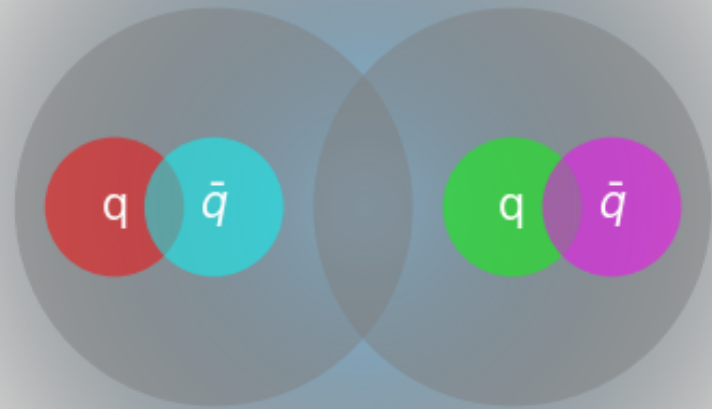


# Exotic states

Instead of baryons and mesons we can have more exotic hadrons

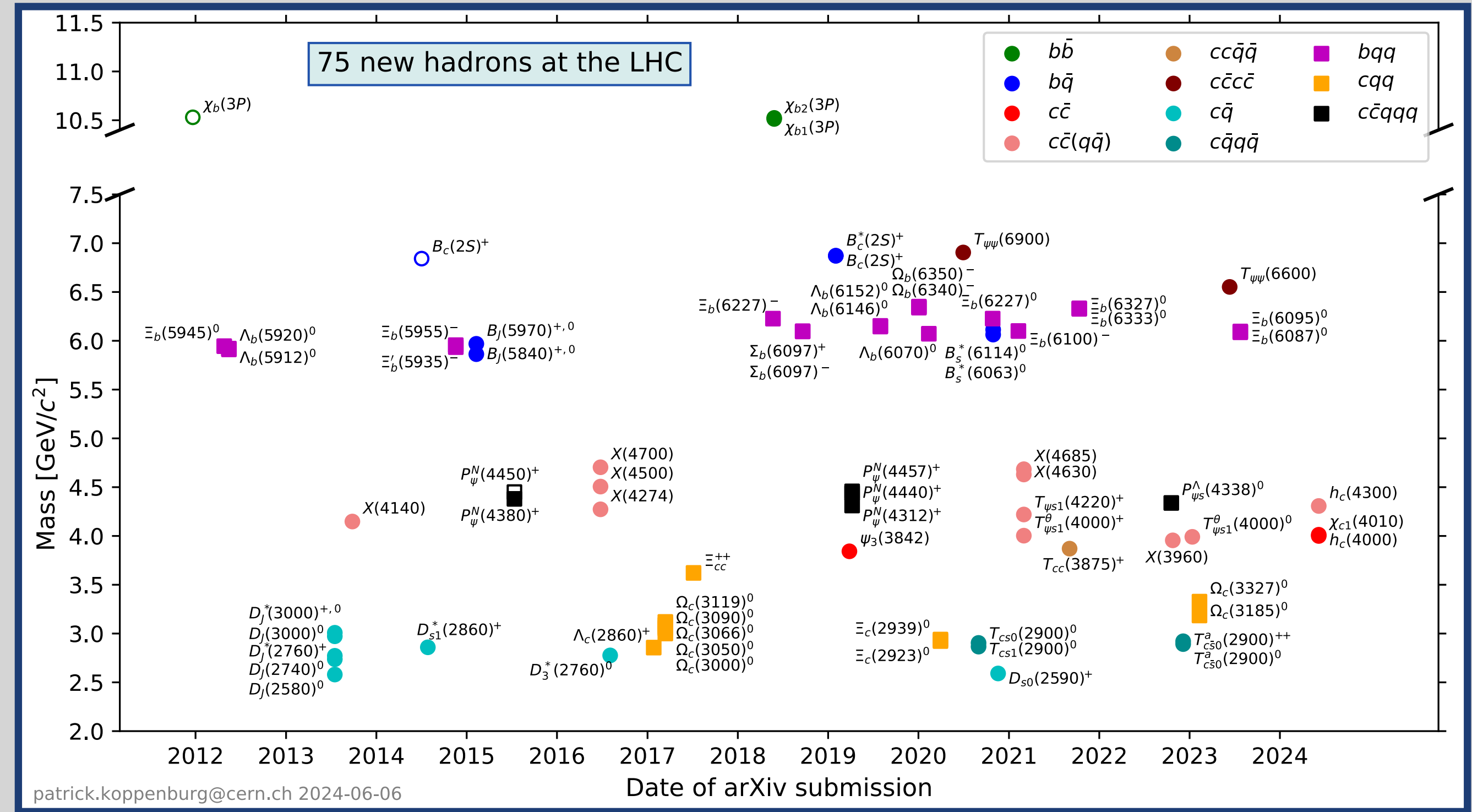
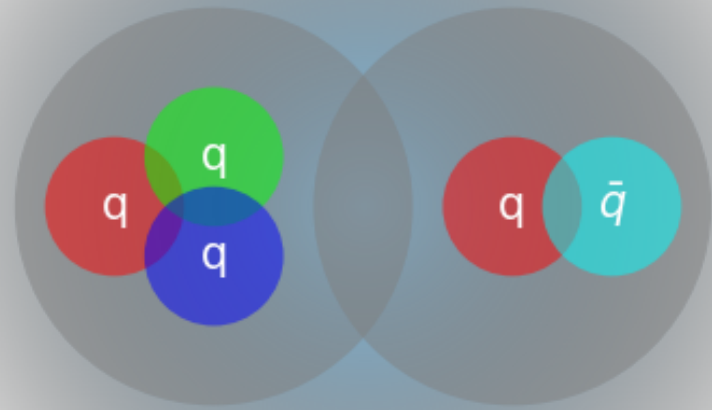
Predicted in quark model

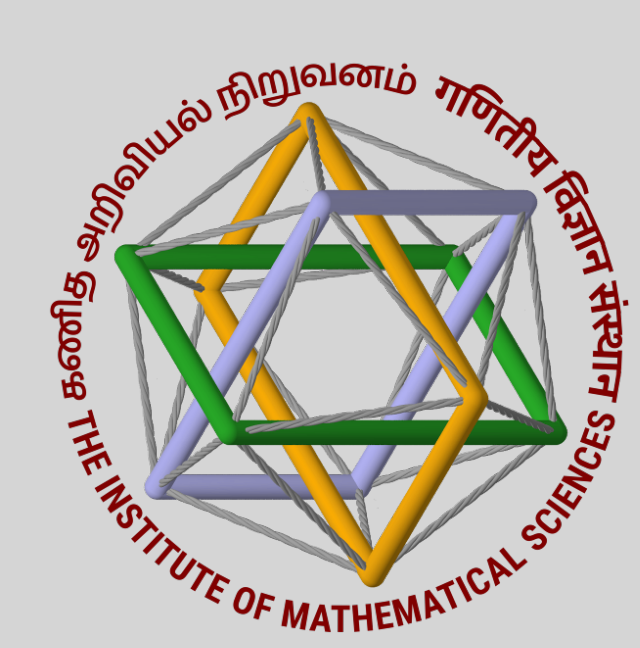
Recent experimental results show their existence



Hadron with 4 quarks

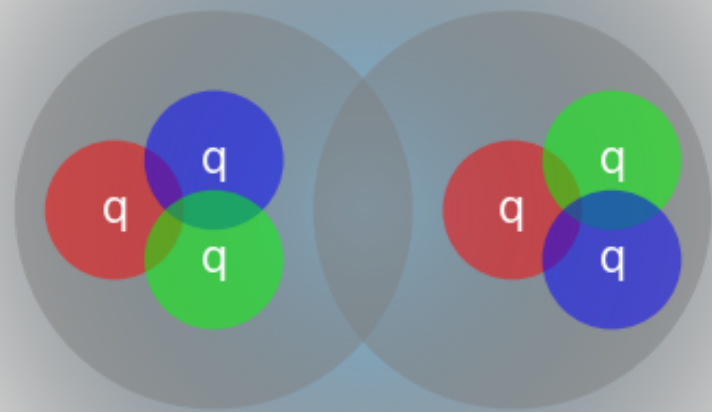
Hadron with 5 quarks





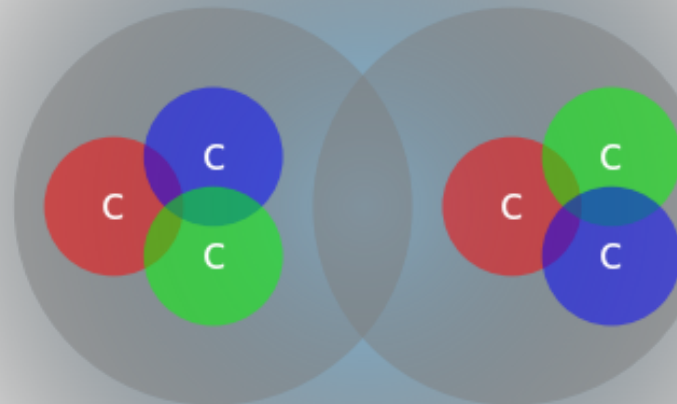
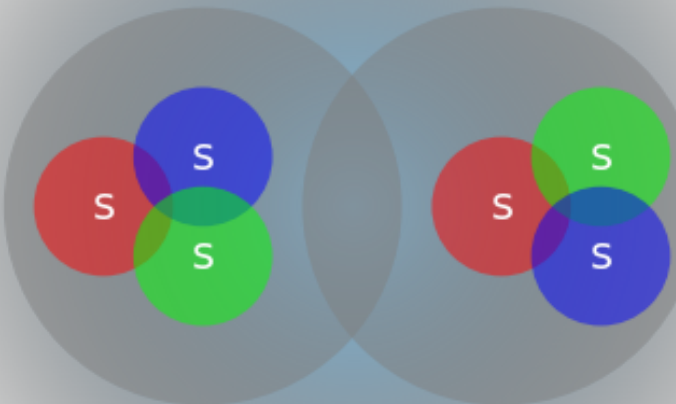
# Hexaquark - Dibaryon

Extensive studies of deuteron like heavy dibaryons using Lattice QCD

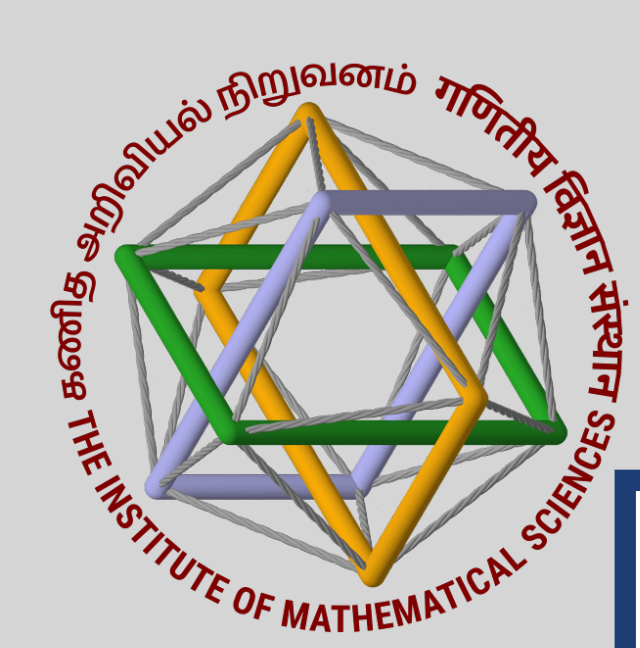


Hadron with 6 quarks

Deuteron



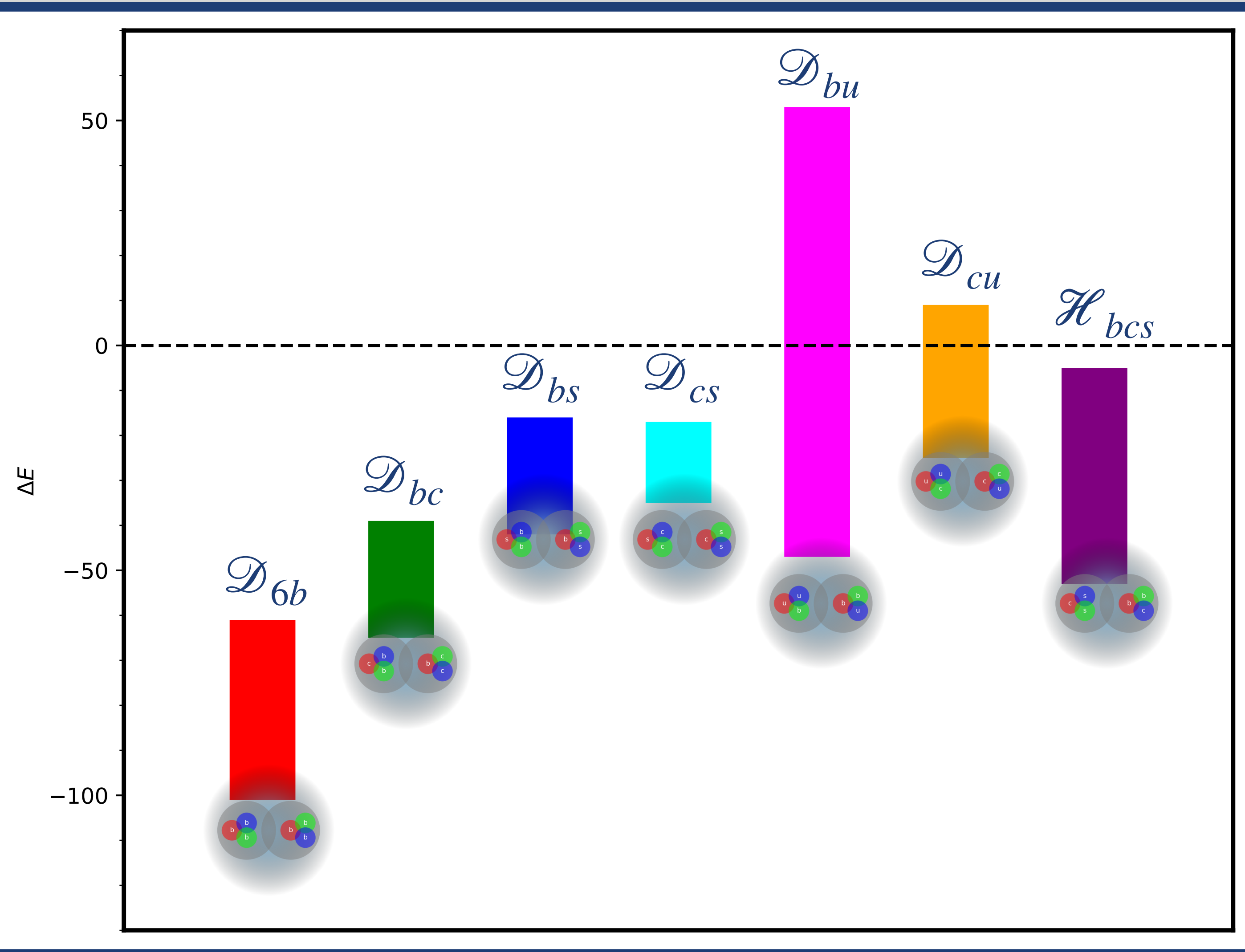
We work with single flavored dibaryons composed of strange and charm quarks named as  $\mathcal{D}_{6s}$  and  $\mathcal{D}_{6c}$  respectively



# Dibaryons results from Lattice

$$D_{6b}, S = 0$$

Mathur, Padmanath, Chakraborty  
PRL, 130, 111901 (2023)



$$D_{bc}, D_{bs}, D_{cs}, D_{bu}, D_{cu}$$

$$S = 1$$

Junnarkar, Mathur PRL, 123, 162003 (2019)

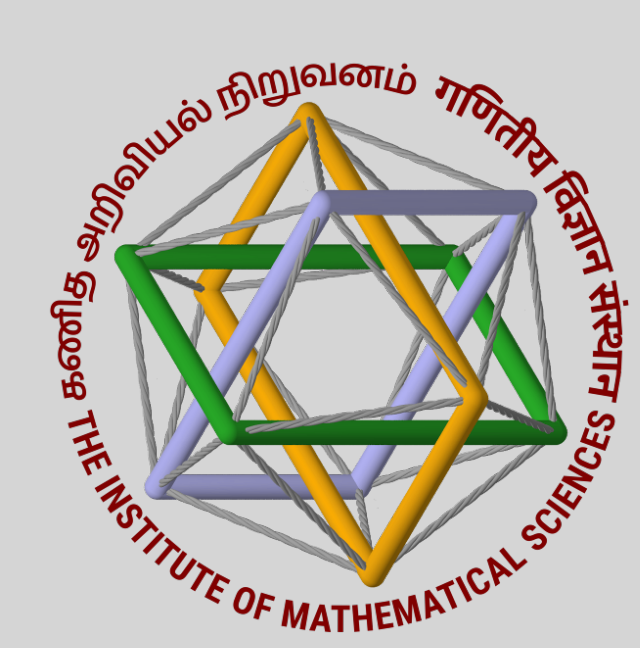
$$\mathcal{H}_{bcs}, S = 0$$

Junnarkar, Mathur PRD, 106, 054511 (2022)

Where does  $D_{6c}, D_{6s}$  stand ??

$$\Delta E = E_{\text{dibaryon}} - E_{\text{1st baryon}} - E_{\text{2nd baryon}}$$





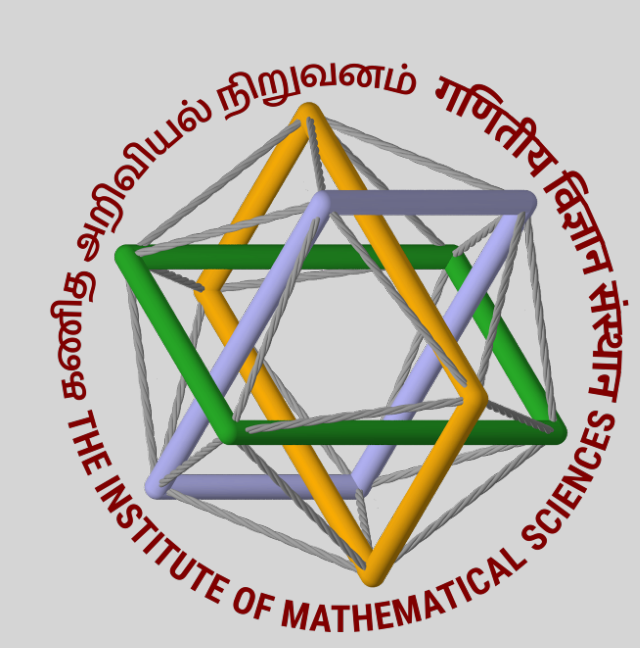
# Dibaryon Operators

$$\mathcal{O} = \epsilon_{abc} q_{\mu_1}^a q_{\mu_2}^b q_{\mu_3}^c$$

$$\mathcal{O}_d = \mathcal{O}_1 \cdot CG \cdot \mathcal{O}_2$$

- Total wave function anti-symmetric under exchange of baryons.
- Single flavor baryons (symmetric).
- Assume only s wave interactions (symmetric) in dibaryon system.
- Color singlet baryons (symmetric)
- Hence Spin must be anti-symmetric which is in case of even spin (Spin 0 and 2)

- Use reduction coefficients to project continuum based operators to suitable octahedral group.
- $S = 0$  continuum spin subduces to one dimensional  $A_1^+$  irrep.
- $S = 2$  continuum spin subduces to two dimensional  $E^+$  and three dimensional  $T_2^+$  irrep.



# Dibaryon Operators

a,b - different embeddings

$$\mathcal{O}_{d,A_1,1}^{[0]} = \frac{1}{2} \left( {}^a H_{3/2} {}^b H_{-3/2} - {}^a H_{1/2} {}^b H_{-1/2} + {}^a H_{-1/2} {}^b H_{1/2} - {}^a H_{-3/2} {}^b H_{3/2} \right)$$

For Spin 0, dibaryon operator corresponding to one dimensional  $A_1^+$  irrep.

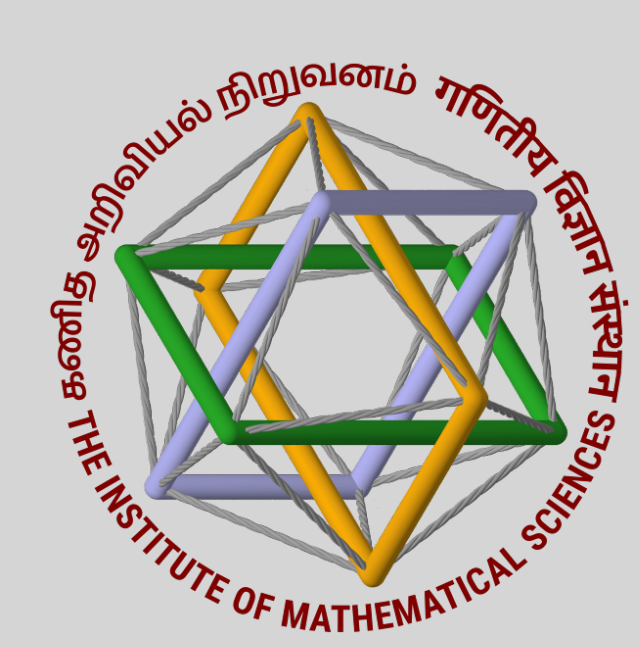
For Spin 2, 5 such operators corresponding to  $E^+$  and  $T_2^+$  irrep.

$S_z$	Operator	State
3/2	${}^1 H_{3/2}$	111
1/2	${}^1 H_{1/2}$	112+121+211
-1/2	${}^1 H_{-1/2}$	122+212+221
-3/2	${}^1 H_{-3/2}$	222

Non Relativistic Embedding

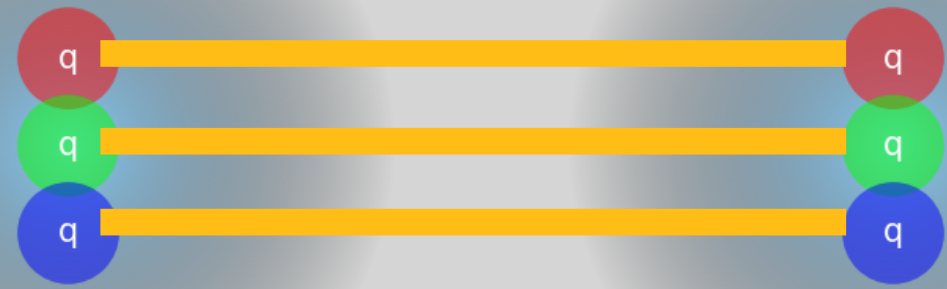
$S_z$	Operator	State
3/2	${}^2 H_{3/2}$	133+313+331
1/2	${}^2 H_{1/2}$	233+323+332+134+341+413+143+431+314
-1/2	${}^2 H_{-1/2}$	144+414+441+234+342+423+243+432+324
-3/2	${}^2 H_{-3/2}$	244+424+442

Relativistic Embedding

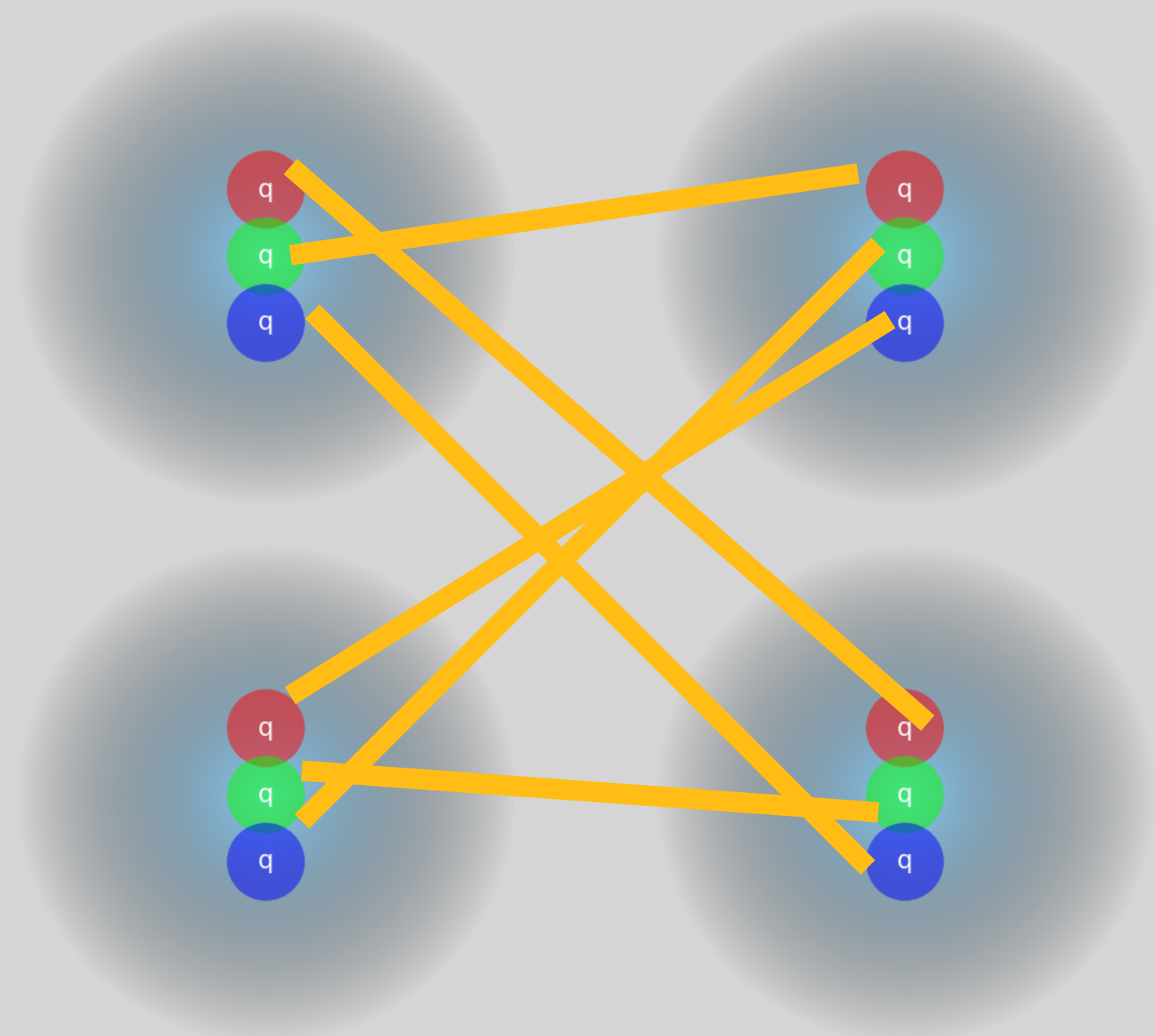


# Operator Contraction

Now we have two baryons at source and two at sink



$6! = 720$  contraction possibilities

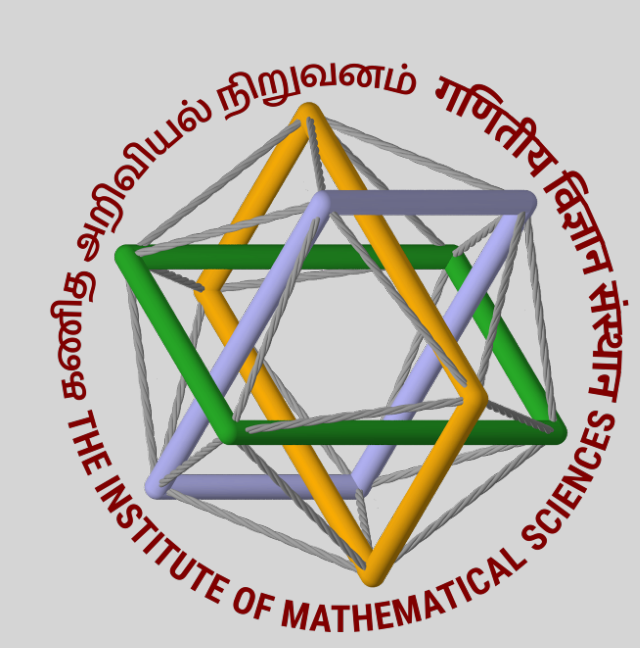


720 contractions can happen in 16 different ways depending on different embeddings

N-N-N-N	N-N-N-R	N-N-R-N	N-N-R-R
N-R-N-N	N-R-N-R	N-R-R-N	N-R-R-R
R-N-N-N	R-N-N-R	R-N-R-N	R-N-R-R
R-R-N-N	R-R-N-R	R-R-R-N	R-R-R-R

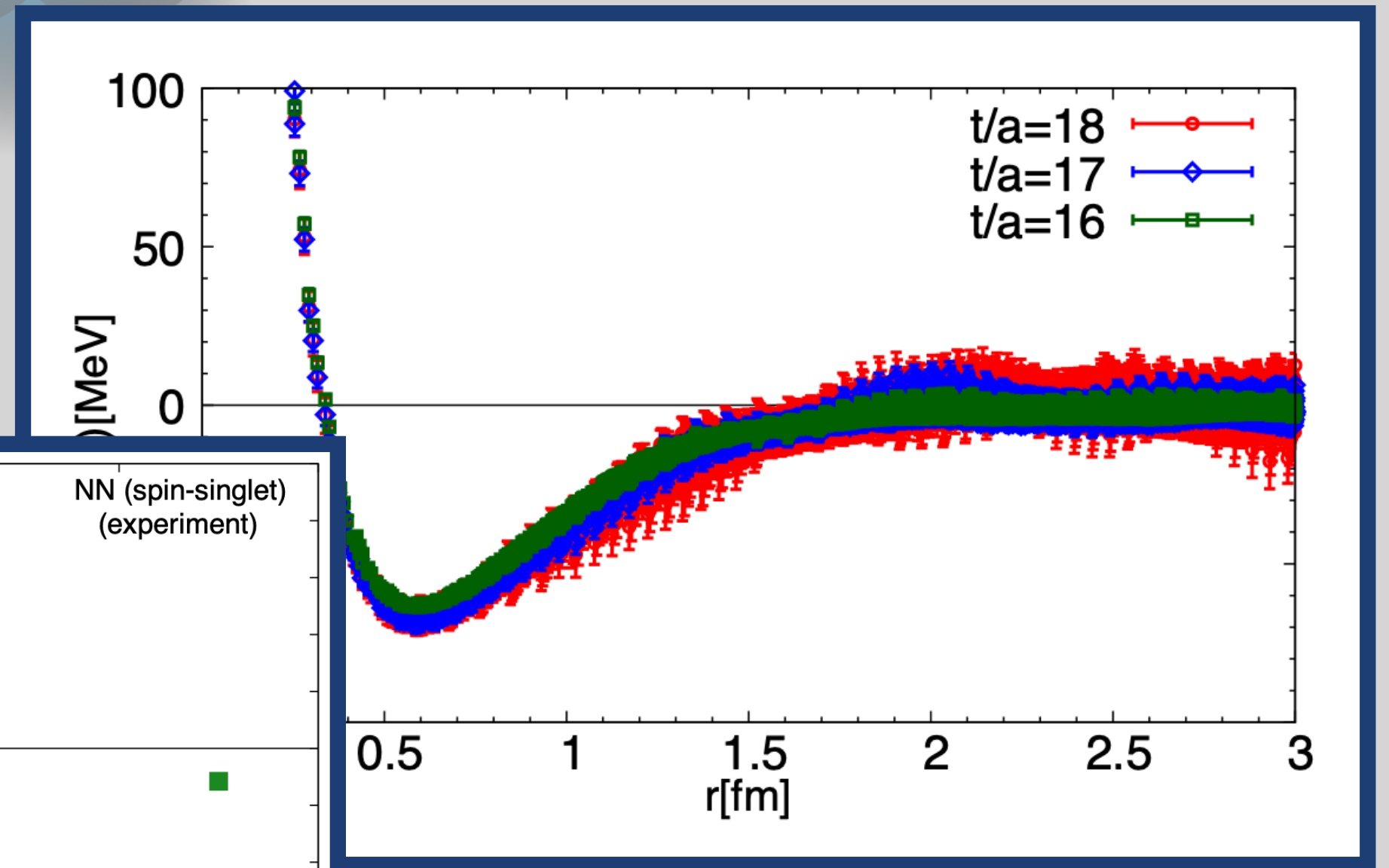
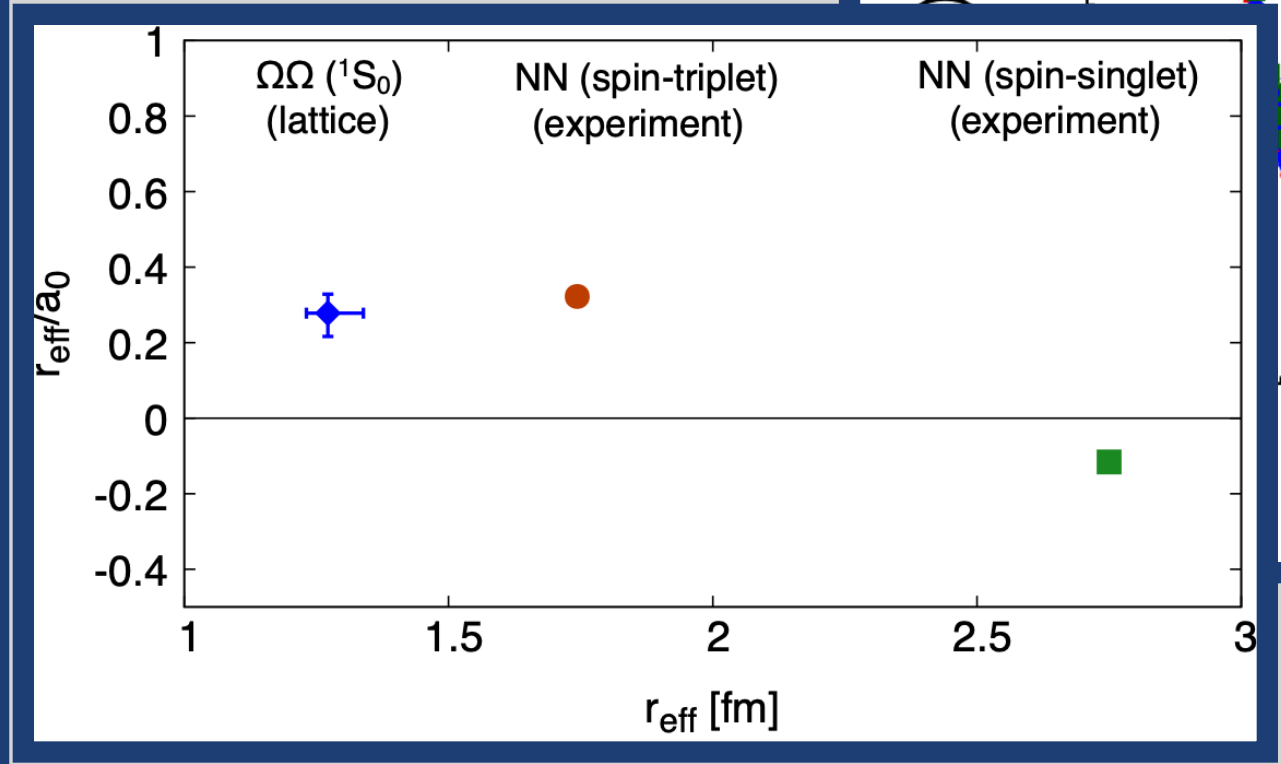
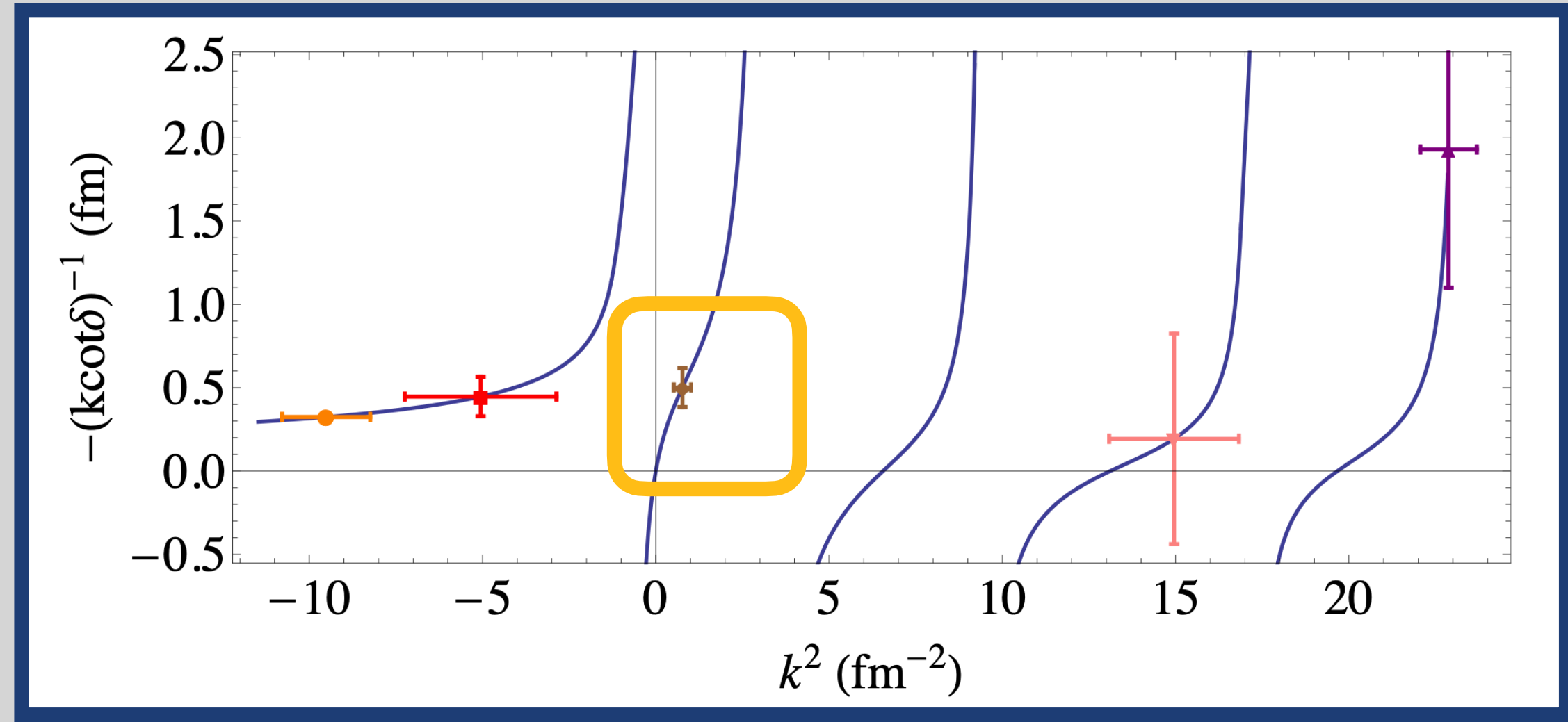
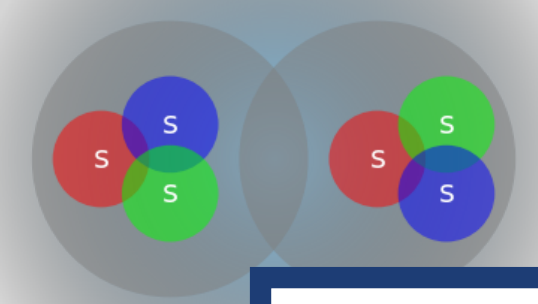
Some of these are degenerate and some of these do not contribute at all for different Spin cases





# $D_{6s}$ Existing Results

HALQCD  
PRL 120, 212001 (2018)

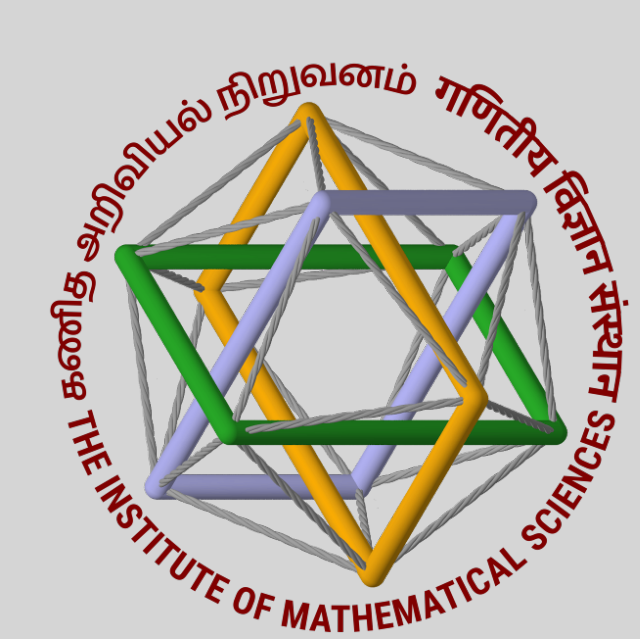


Buchoff, Luu, Wasem  
PRD 85, 094511 (2012)

- Weakly repulsive in Spin 0  $H^+H^+$  irrep, No bound state
- Attractive in Spin 1,2  $G_1^+H^+$  but only single volume used.

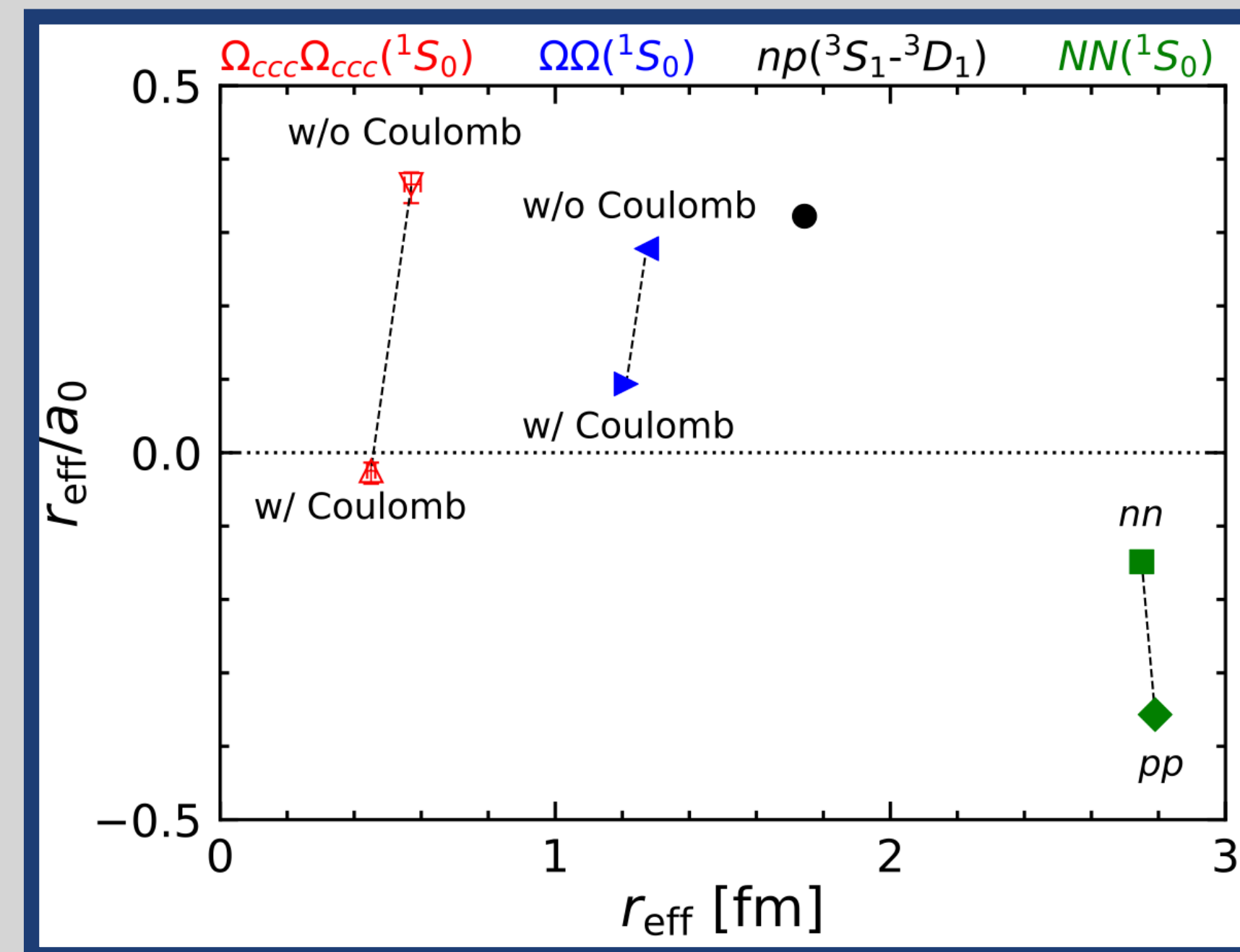
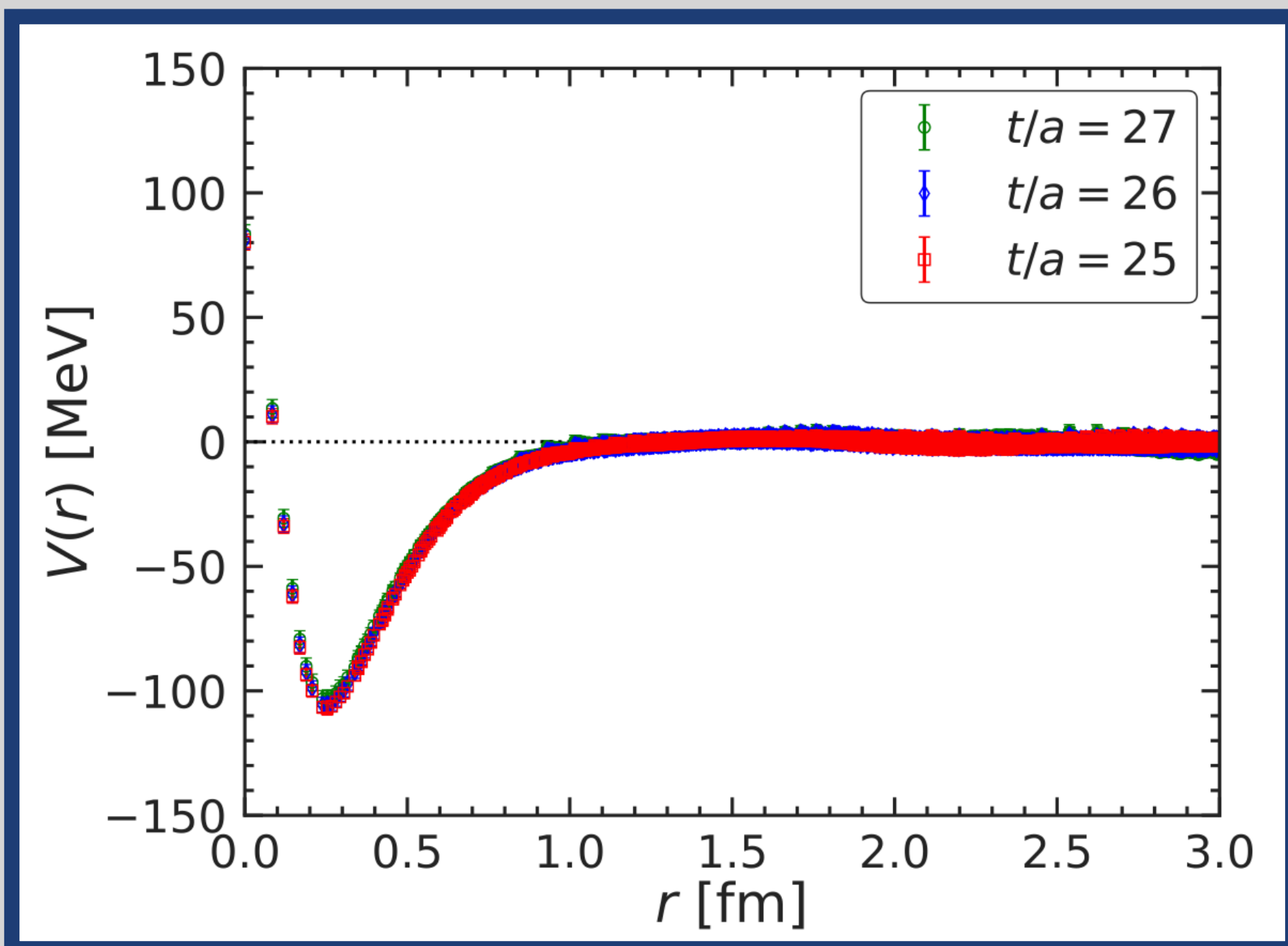
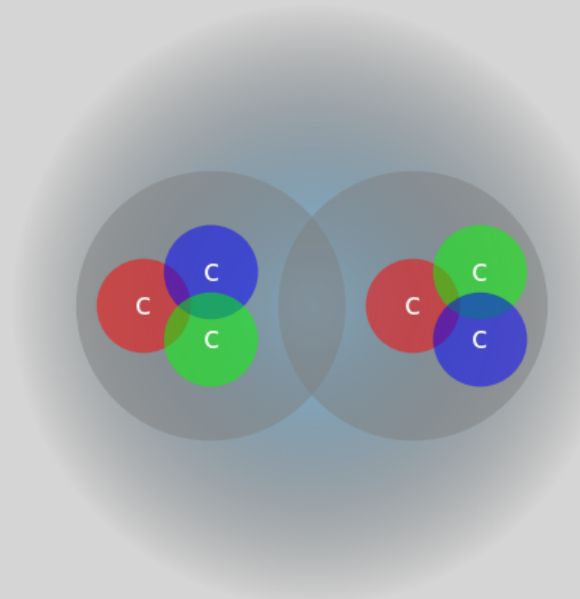
- Weakly attractive in Spin 0, hence bound state
- “Such a system can be best searched experimentally by the pair-momentum correlation in relativistic heavy-ion collisions.”



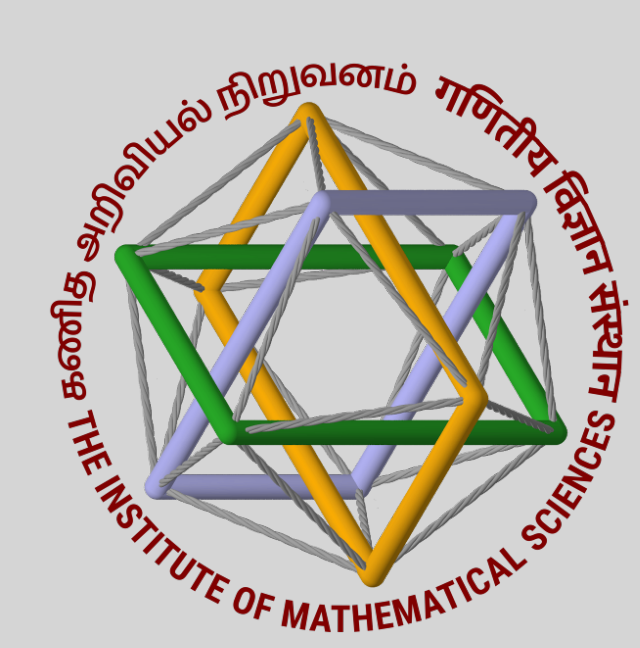


# $D_{6c}$ Existing Results

HALQCD  
PRL 127, 072003 (2021)



- Simulation with physical charm mass and near physical light quark mass.
- Dibaryon existence without Coulomb interaction.
- Near unitary region with Coulomb interaction (scattering length less than corresponding strange dibaryon calculation).



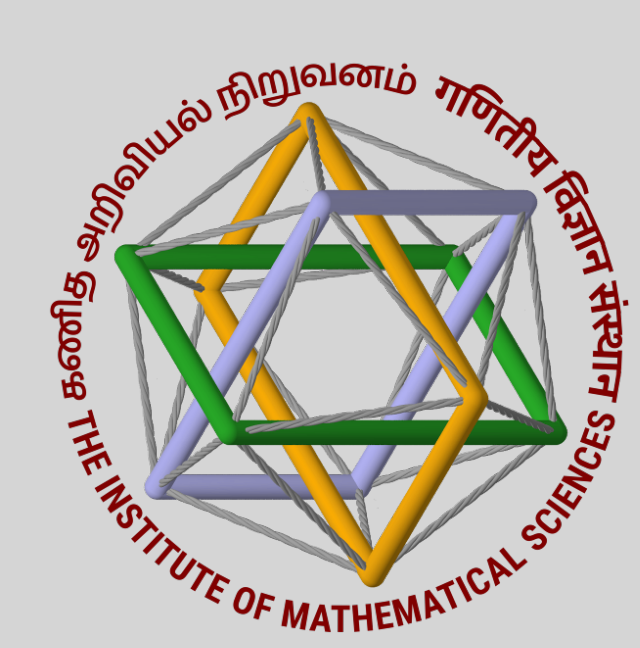
# Is this a bound state?

- ★ Quoting bound state and binding energies directly from energy splittings of product hadron and its constituents.
- ★ Extract hadron-hadron interactions by solving QM potentials from Nambu-Bethe-Salpeter wave function.
- ★ Luscher's formalism - relating discrete finite-volume energy spectrum to few-body scattering amplitudes in the infinite limit.

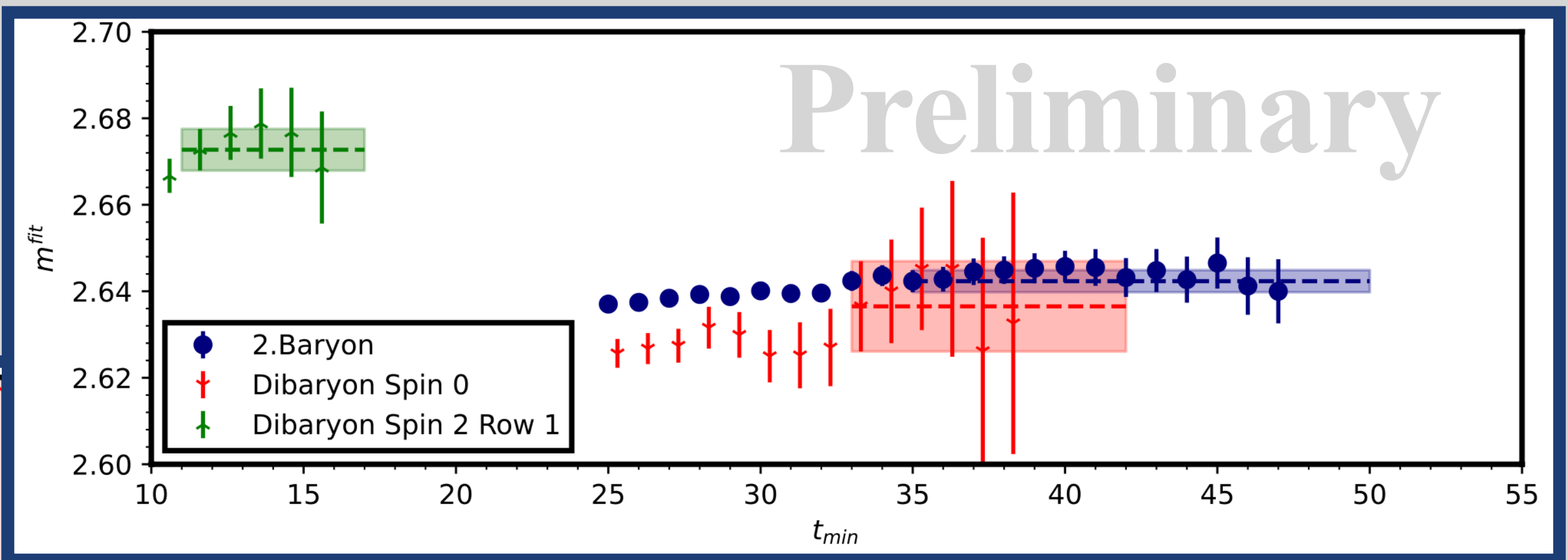
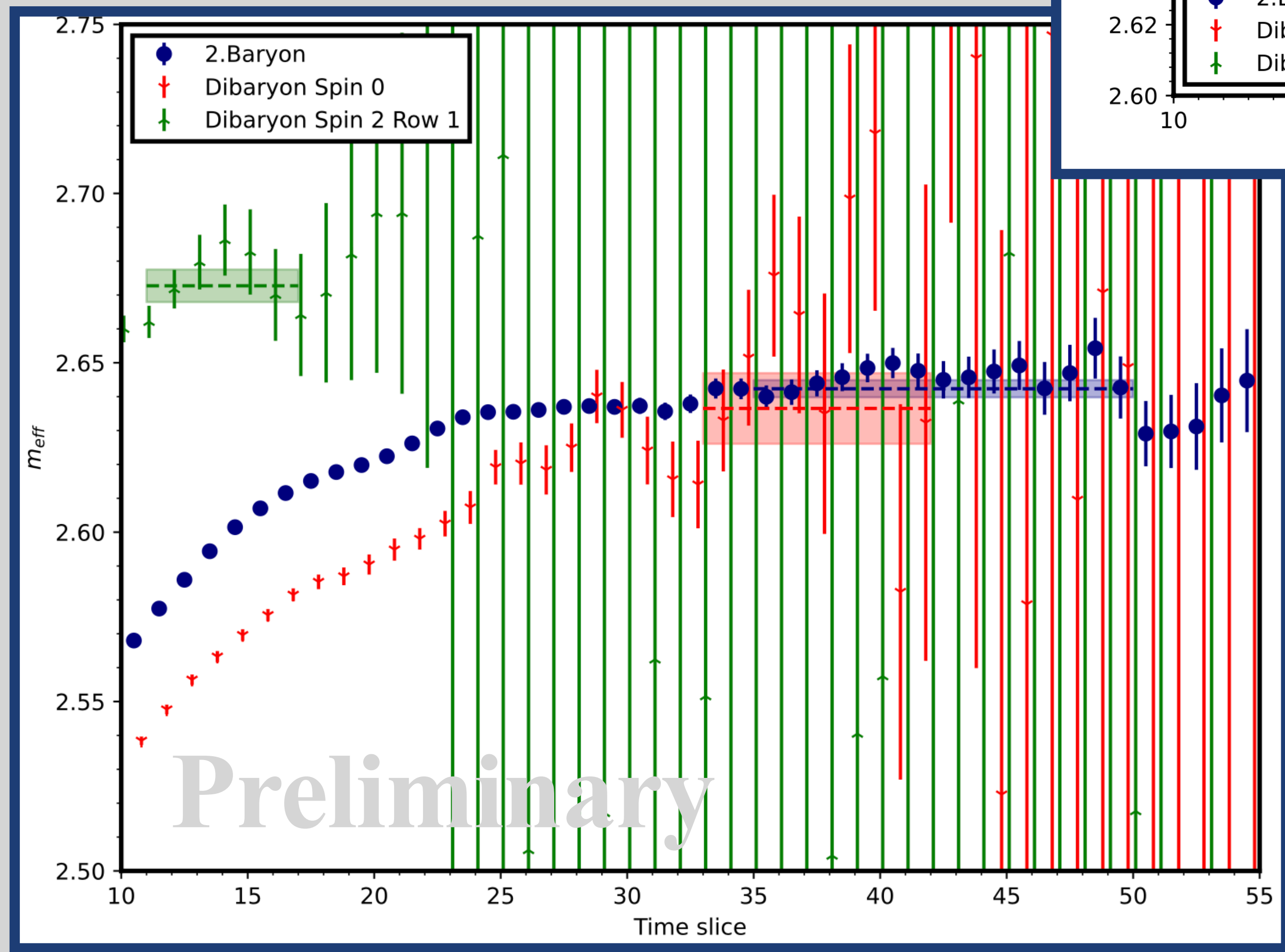
In this talk

Previous slide  
HALQCD

Near future

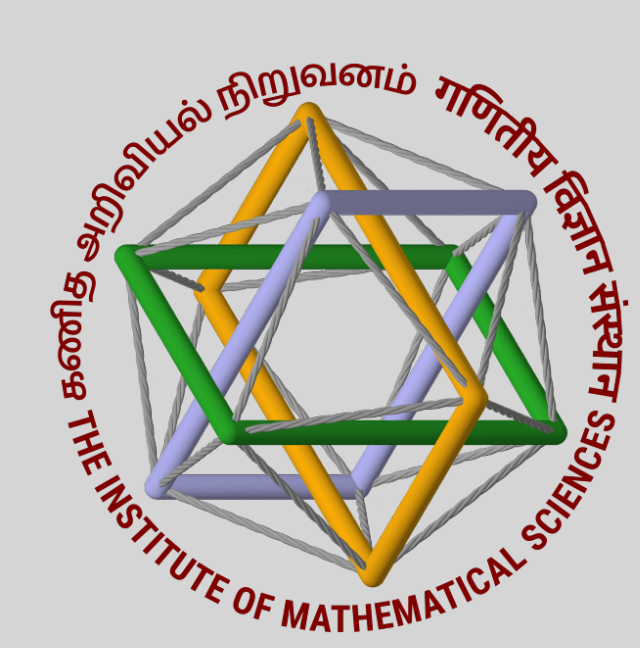


# Energy spectrum

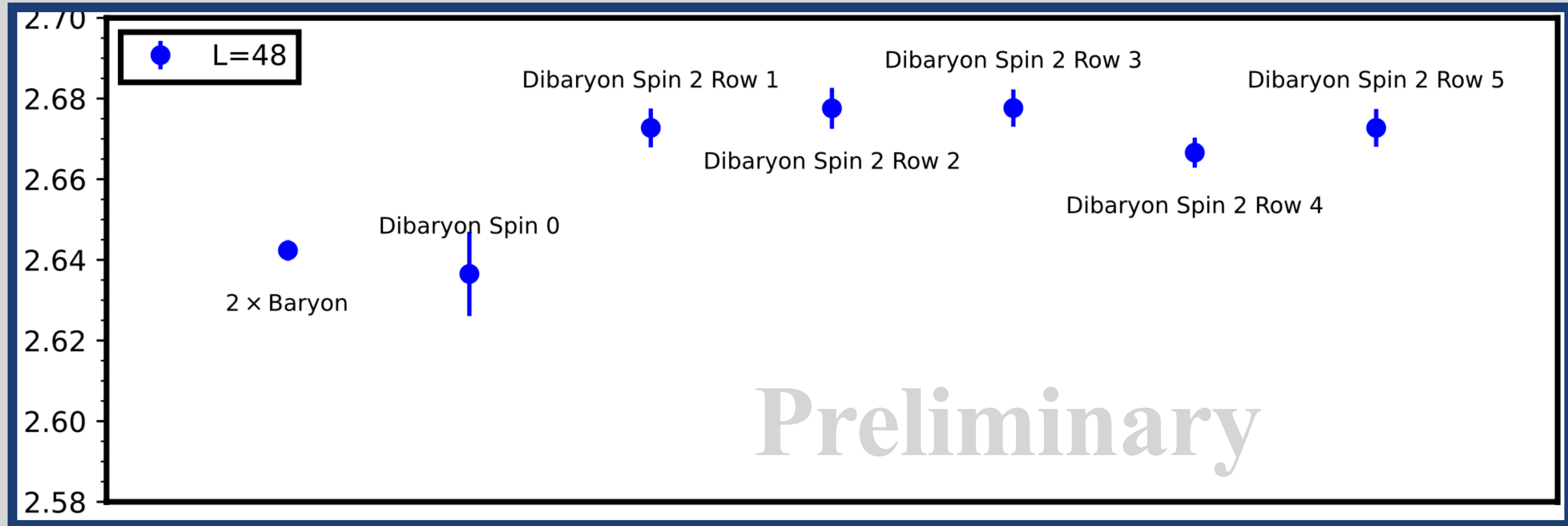
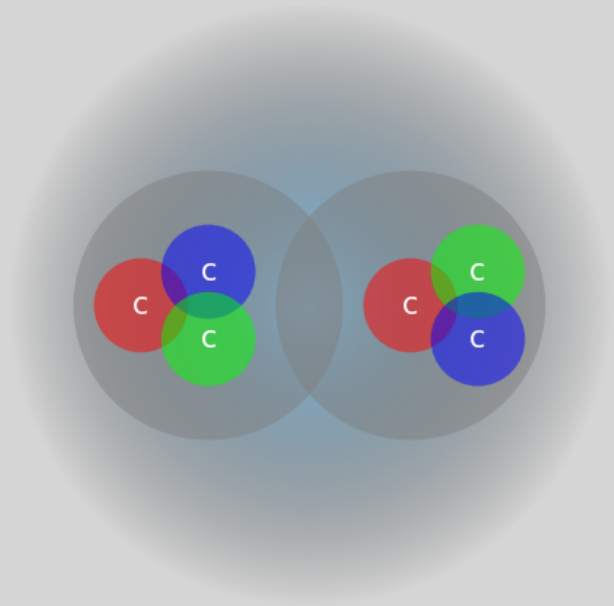


- Dependence of minimum time slice for fitting.

- Effective masses from lattice by using Dibaryon Spin 0 operator, one operator of Spin 2 for Dibaryon.
- Comparison with twice the effective mass from Baryon operator.



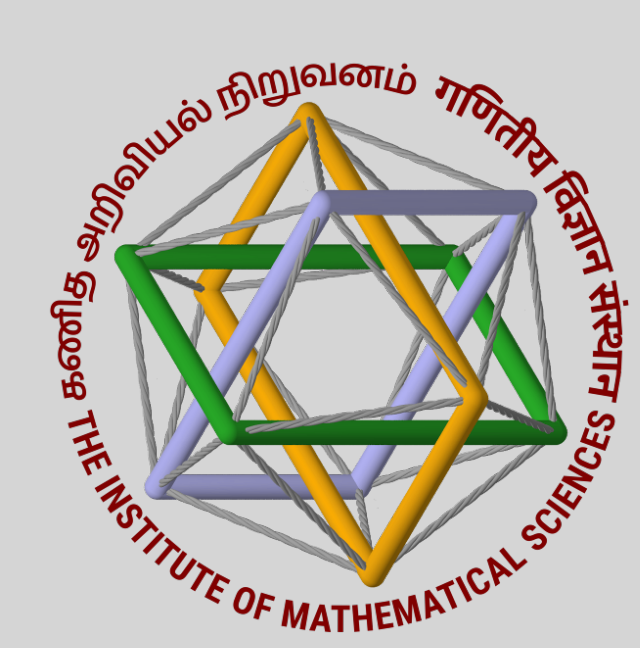
# $\mathcal{D}_{6c}$ Results



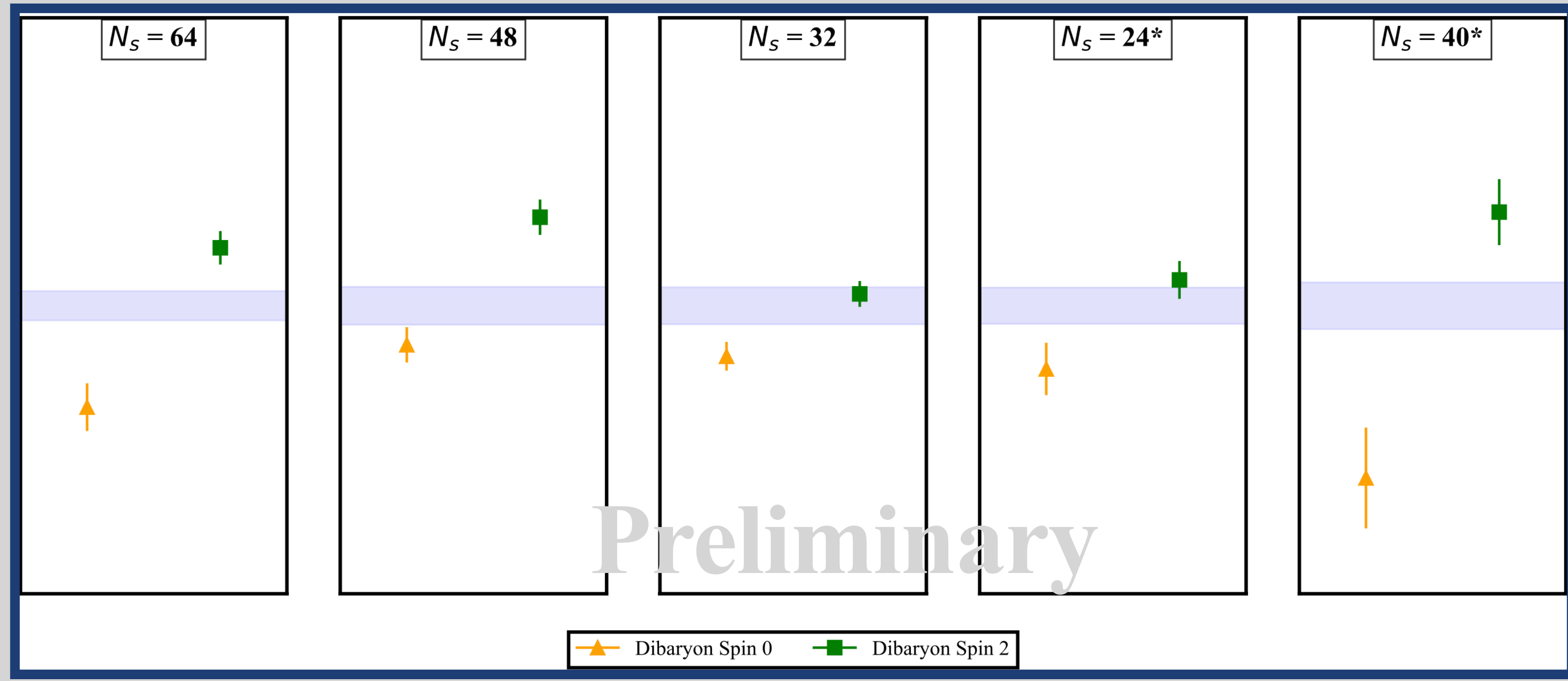
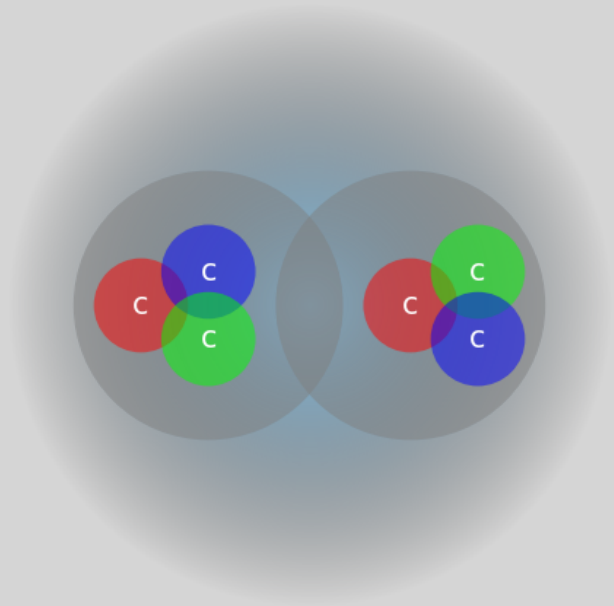
- Bound state, if exist, is shallow
- More probe using Luscher's formalism

- Two lattice volumes, 4 lattice spacings, this plot with  $L = 48$
- Spin 2 - repulsive interactions, Spin 0 dibaryon energy same as twice of baryon



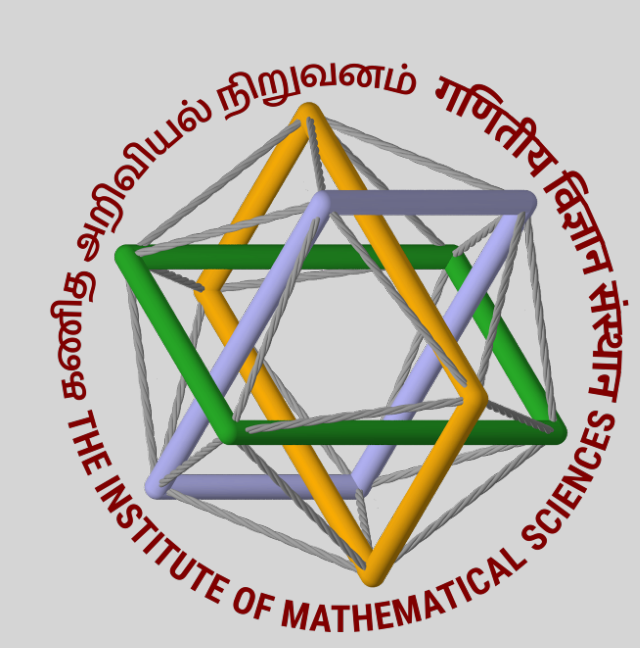


# $D_{6c}$ Results

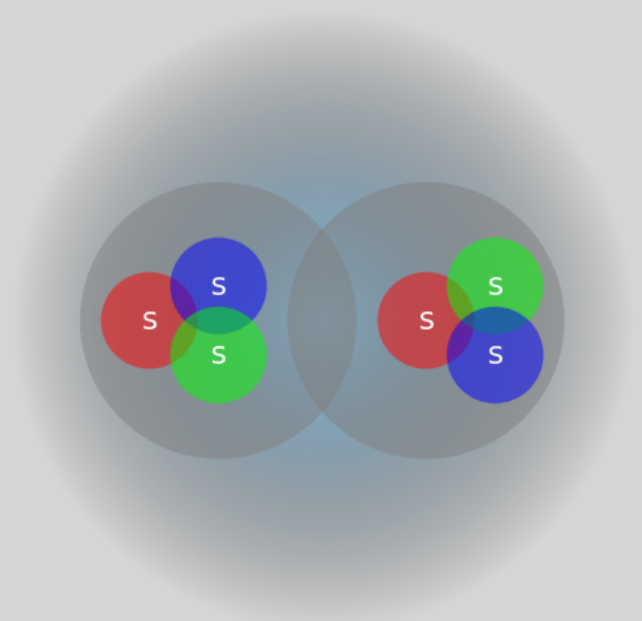


- Bound state, if exist, is shallow
- More probe using Luscher's formalism

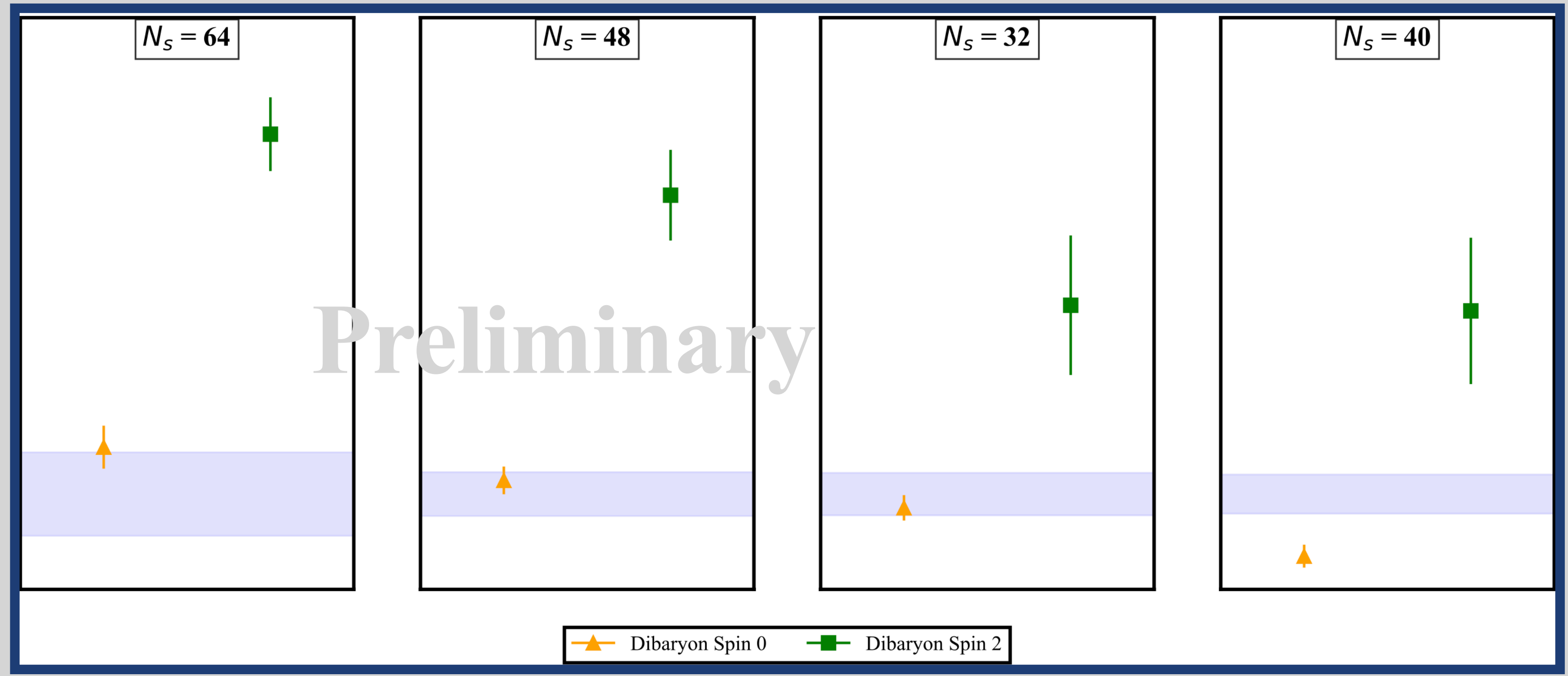
HALQCD  
PRL 127, 072003 (2021)  
Bound state without Coulomb interaction



# $D_{6s}$ Results



- No bound state
- More probe using Luscher's formalism

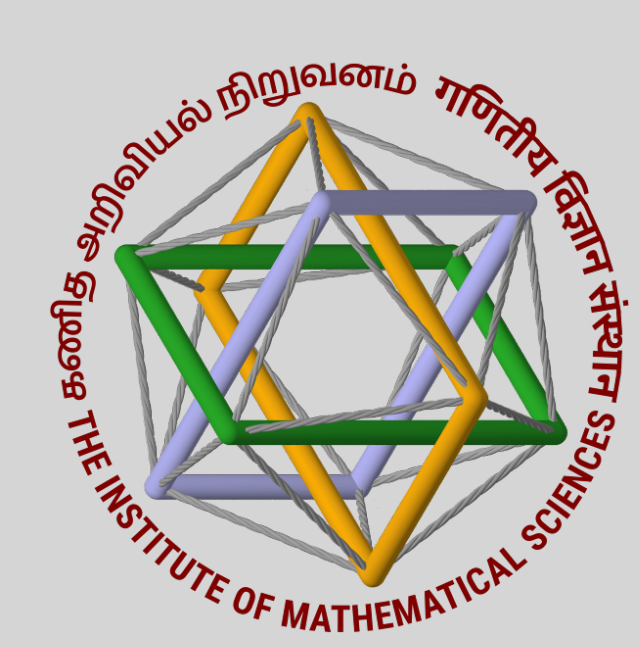


Buchoff, Luu, Wasem  
PRD 85, 094511 (2012)

No bound state

HALQCD  
PRL 120, 212001 (2018)

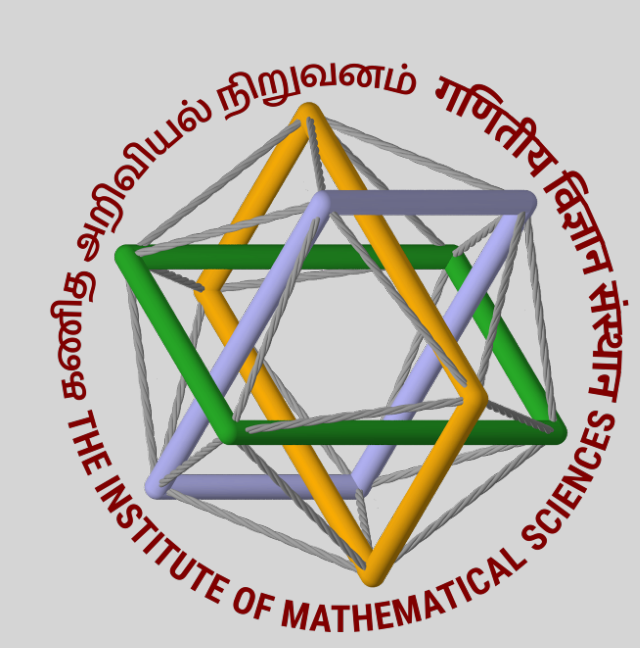
Bound state exists



# Conclusions

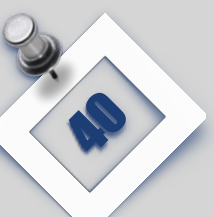
- ★ Simulations with finer and big enough lattices.
- ★  $\Omega$  and  $\Omega_{ccc}$  calculations with more ensembles in the continuum limit.
- ★ Dibaryon investigation in  $\Omega - \Omega$  and  $\Omega_{ccc} - \Omega_{ccc}$  systems.
- ★ Prediction: Absence of bound state (or very weakly bound state if any)





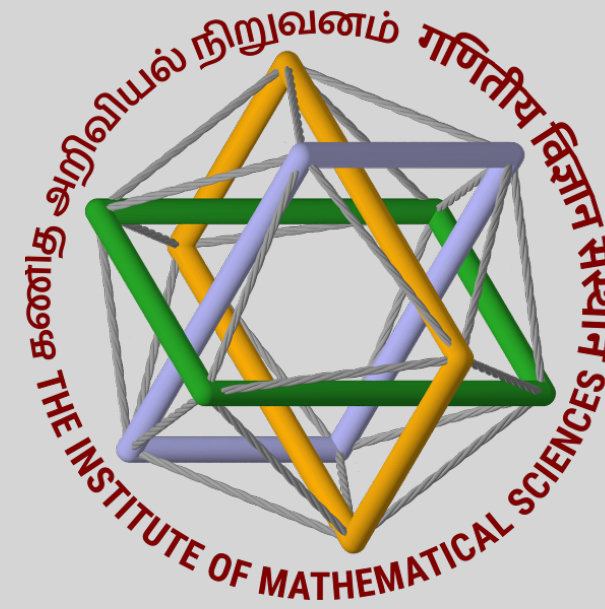
# Future Directions

- ★ Luscher's formalism for dibaryon investigations.
- ★ Initial calculations indicate similar behaviour of  $E^+$  and  $T_2^+$  irrep, more detailed analysis to follow.
- ★ Do the heavier baryons composed of charm and bottom quarks and their corresponding dibaryons (if exist as a bound state) survive with temperature ?
- ★ Lattice estimation of  $d^*(2380)$ ...





# THANK YOU



**Navdeep Singh Dhindsa 15/07/2024**