

EIGENSTATE DIMENSIONALITY IN CHIRAL ANDERSON MODELS: IMPLICATIONS FOR QCD



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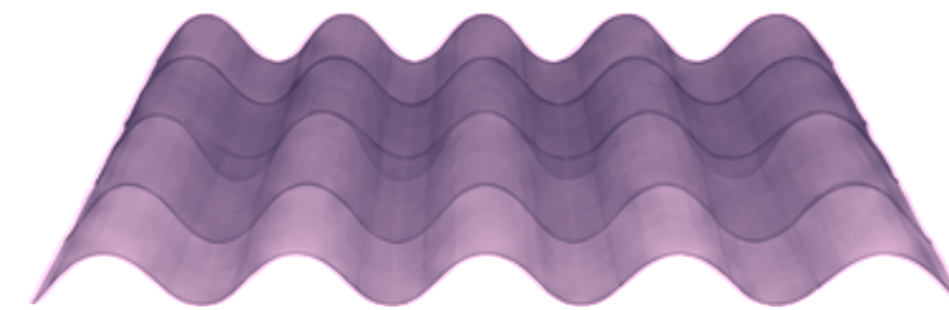
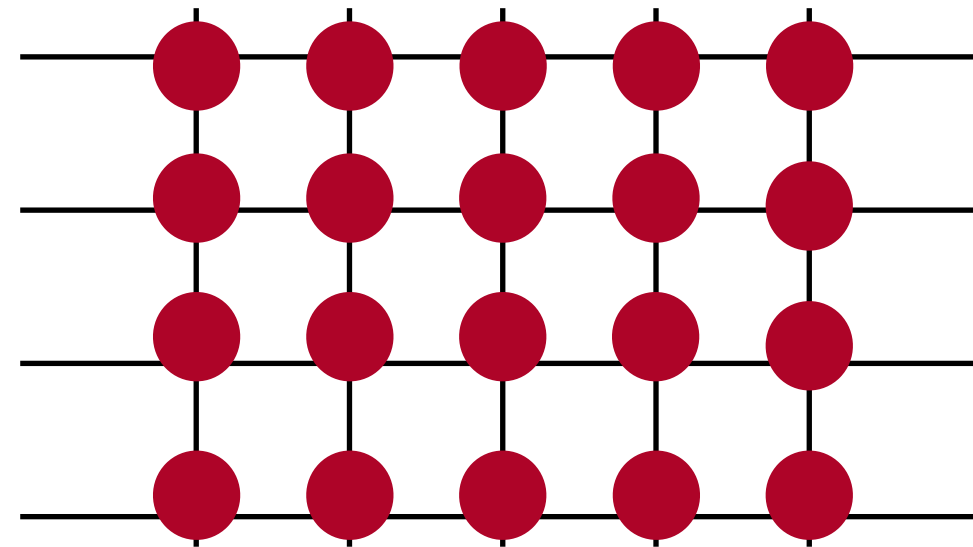
Why are we doing Lattice QCD? For funding applications



Disorder, Localization and Quantum States



Ordered System

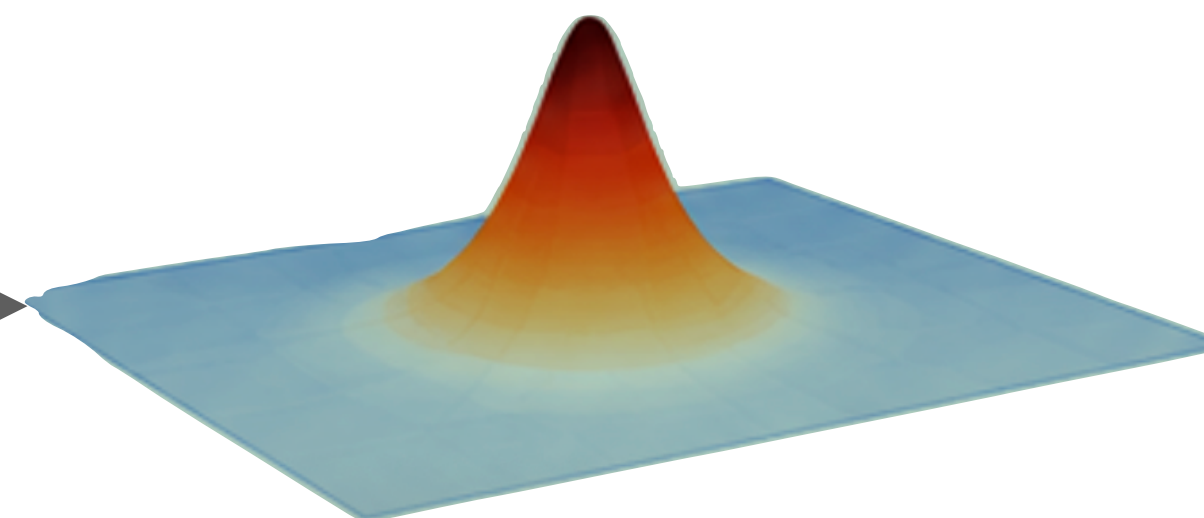
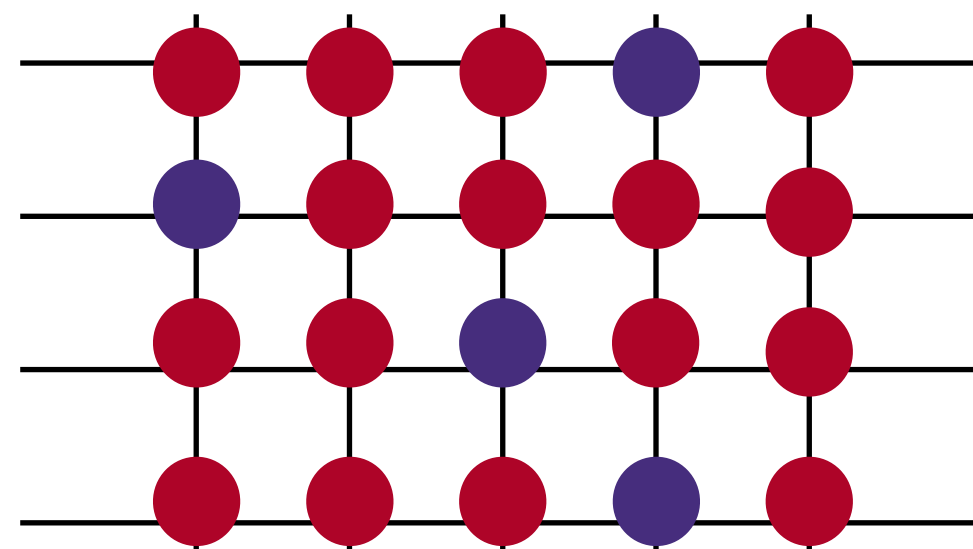


Extended state,
Occupies entire system

► Anderson (1958): Random disorder can localise quantum wavefunctions

► Disorder drives a transition between extended and localised states

Random Disordered System



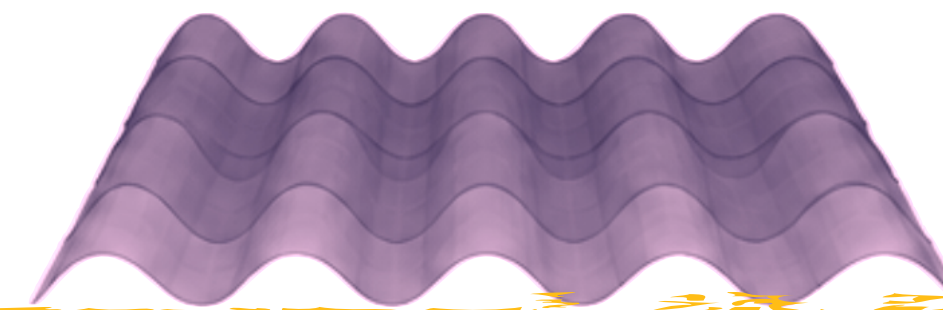
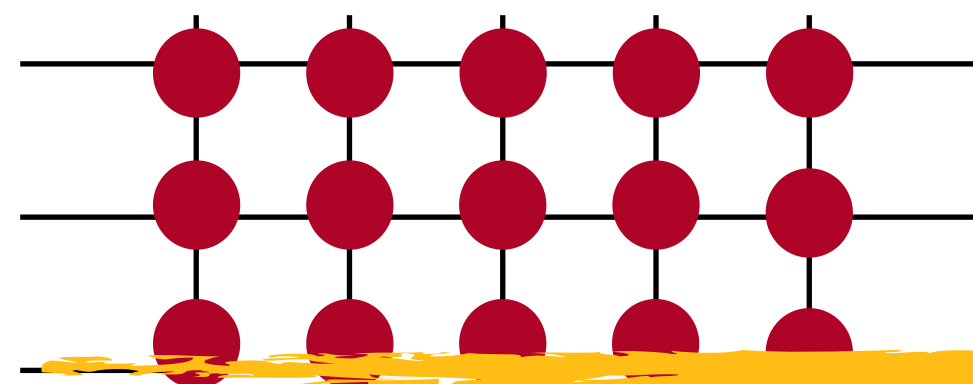
Localized state,
Confined to finite region

► Spatial structure of eigenstates changes dramatically at the transition

Disorder, Localization and Quantum States



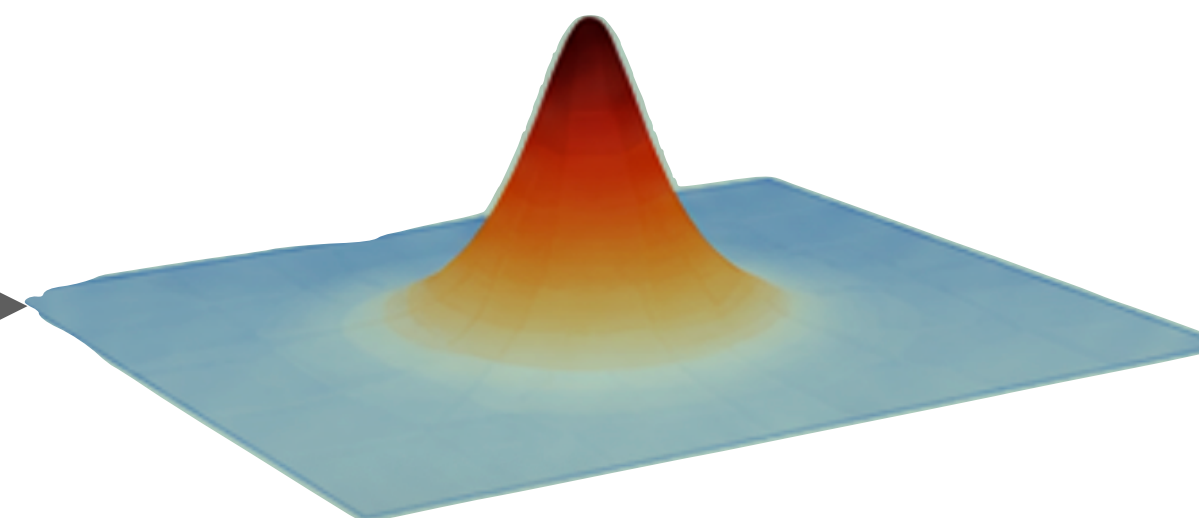
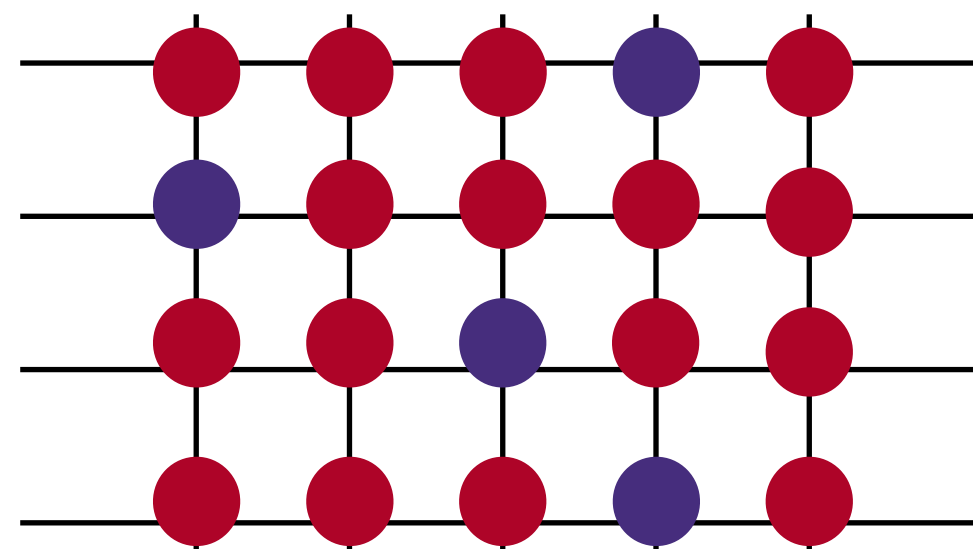
Ordered System



► Anderson (1958): Random disorder can localise quantum wavefunctions

*How does a quantum state occupy space?
Can we quantify its effective dimensions?*

Random Disordered System



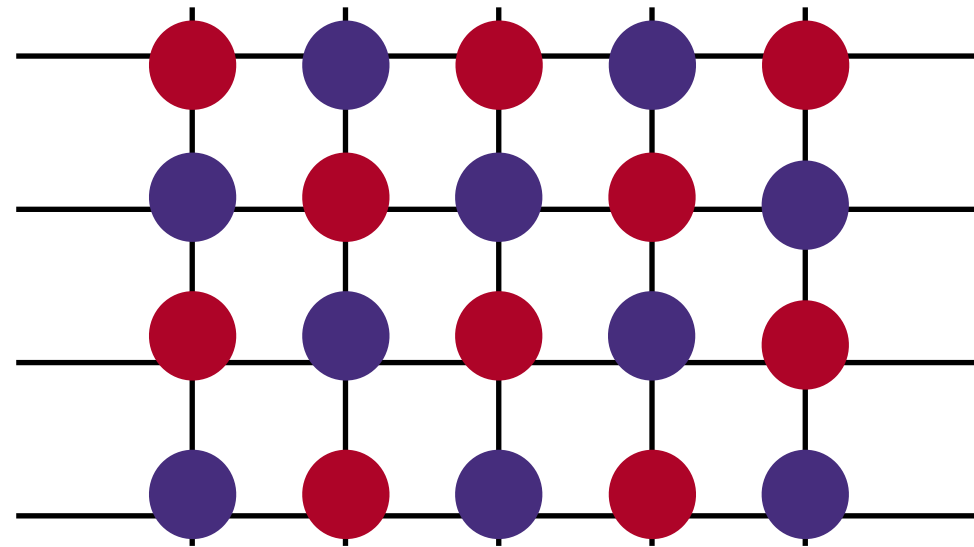
Localized state,
Confined to finite region

► Spatial structure of eigenstates changes dramatically at the transition

Chiral Symmetry



Bipartite Lattice



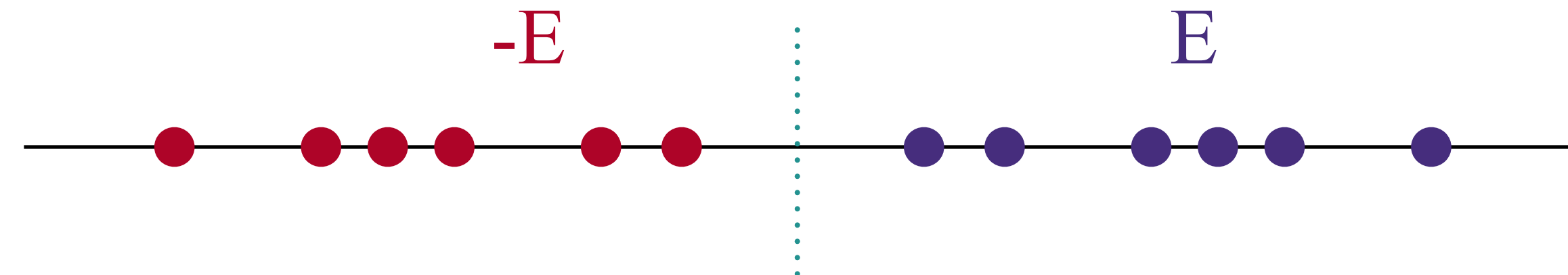
Hopping only between ● and ●

Hence no diagonal disorder

$$H = \begin{pmatrix} 0 & T \\ T^\dagger & 0 \end{pmatrix}$$

Consequences

► Spectrum symmetric around $E = 0$



► Near $E = 0$, DOS develops singular behaviour
(Qualitative Statement!)

► Chiral symmetry introduces a fundamental constraint on the spectrum leading to new localisation phenomenon at $E = 0$.

Why is it relevant?



Quark propagation through a highly fluctuating gluonic background (*disordered medium*)



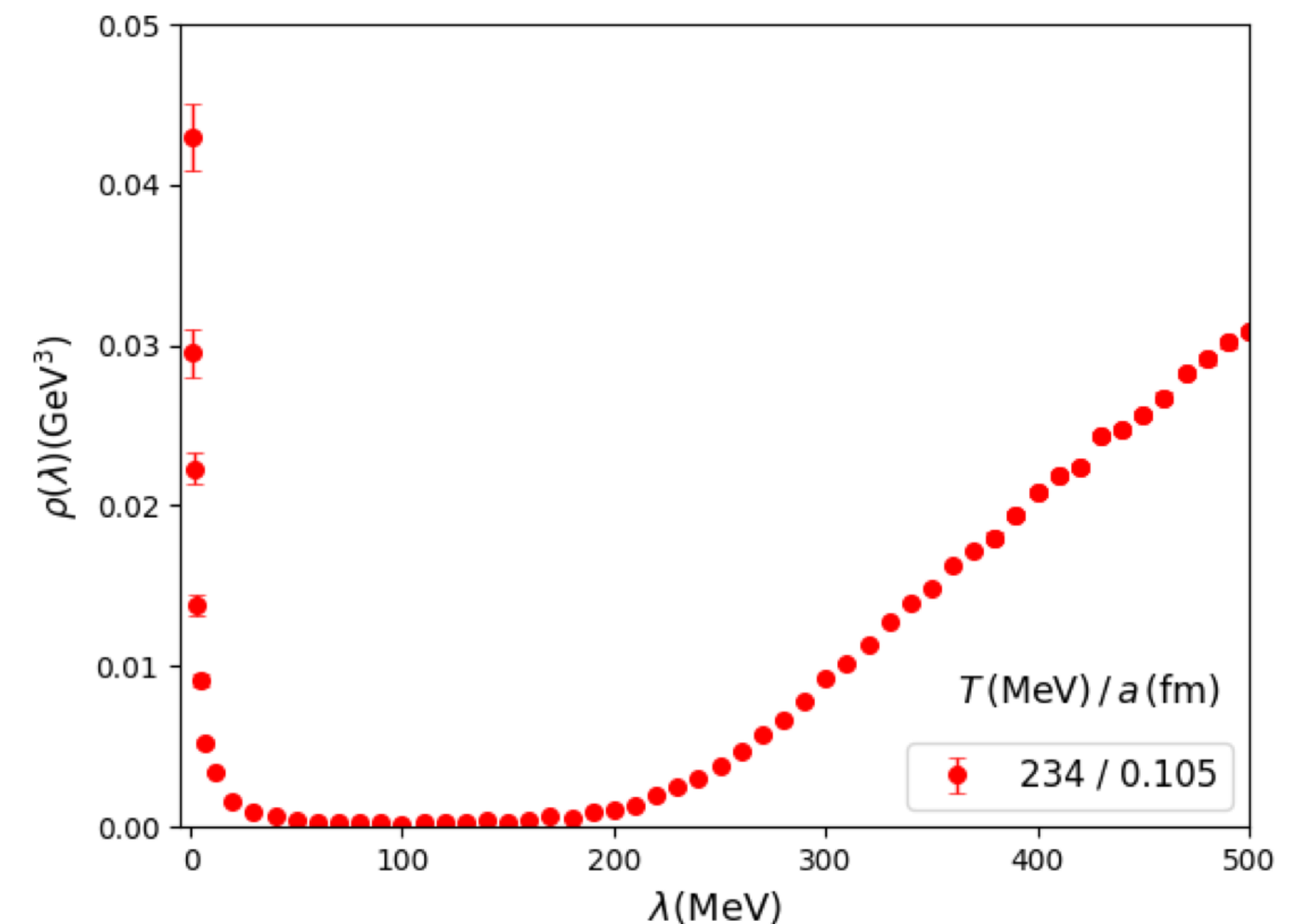
Motion of quark governed by, $D\psi_\lambda = i\lambda\psi_\lambda$



Different gauge configs $\rightarrow$ Different Dirac ops
 $\rightarrow$ Different eigenmodes



Rather than classifying eigenmodes simply as localised or extended, we want to quantify the geometry of the space they occupy



χ QCD and CLQCD
(JHEP 12 (2024) 101)

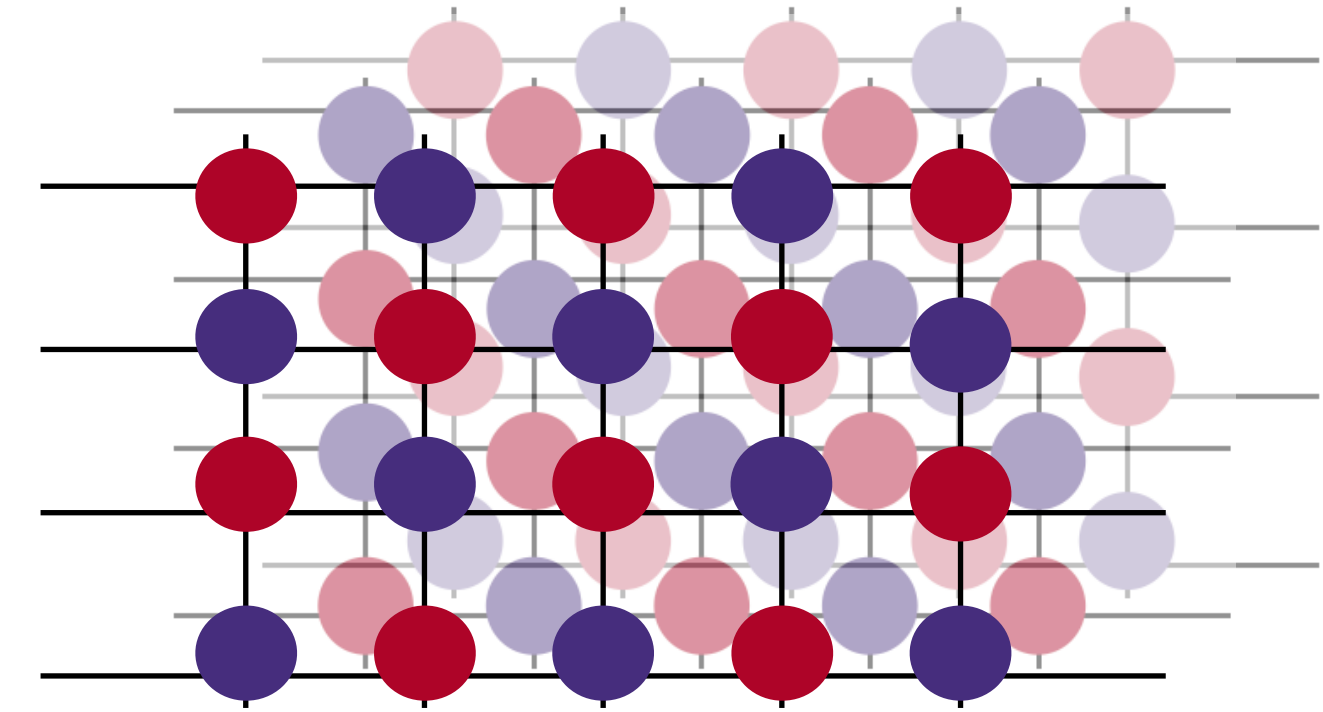
$N_f = 2 + 1$, Clover

Model



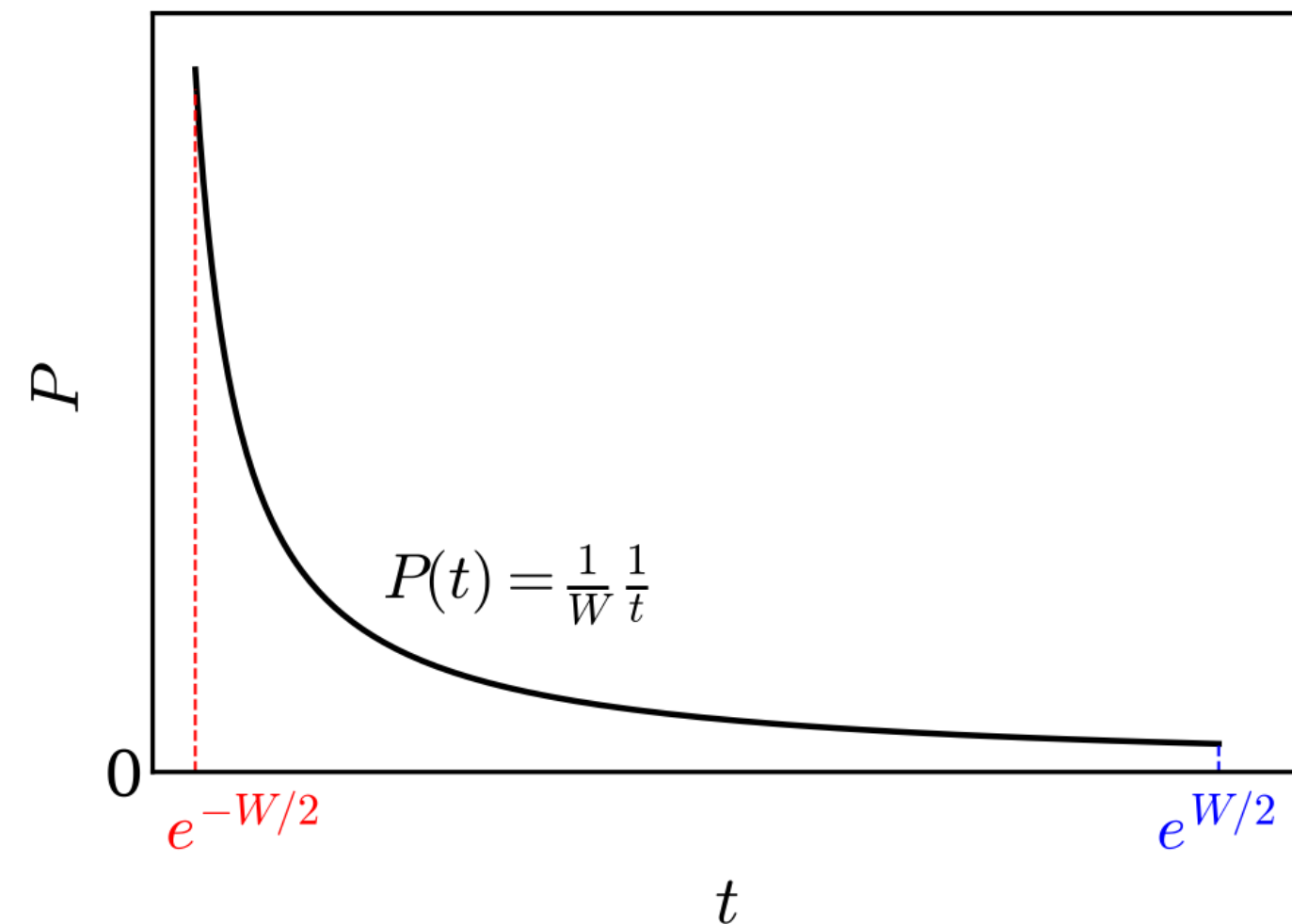
Nearest neighbour hopping only, no on site disorder

$$H_{\mathbf{r},\mathbf{r}'} = \sum_{i=x,y,z} \left[t_{\mathbf{r},\mathbf{r}+\hat{i}} \delta_{\mathbf{r}+\hat{i},\mathbf{r}'} + t_{\mathbf{r},\mathbf{r}-\hat{i}} \delta_{\mathbf{r}-\hat{i},\mathbf{r}'} \right]$$

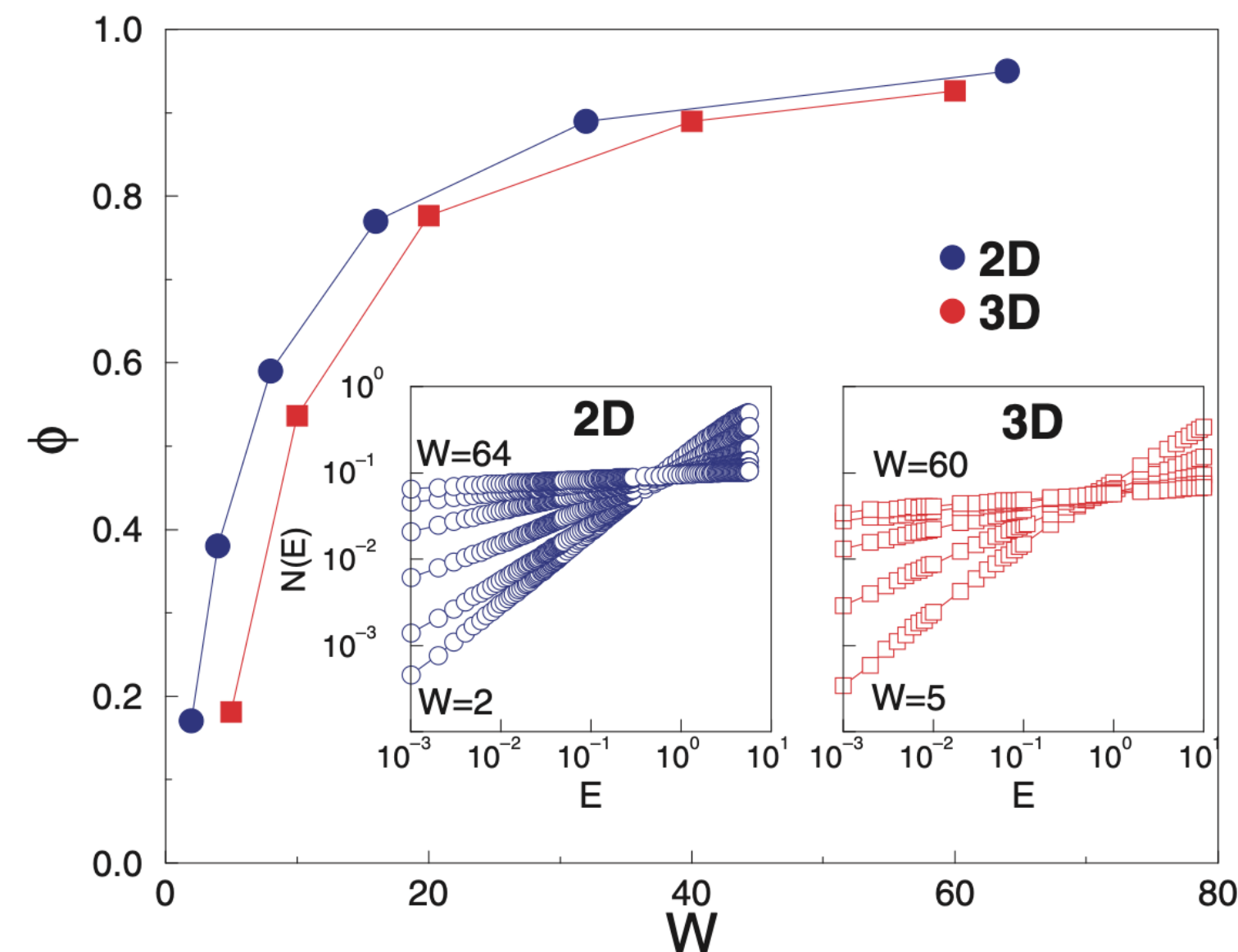


We work with logarithmic disorder

$$t_{\mathbf{r},\mathbf{r}\pm\hat{i}} \in \left[\exp\left(-\frac{W}{2}\right), \exp\left(\frac{W}{2}\right) \right]$$



How with increasing disorder strength W modify geometry and localisation properties of eigenstates?



Exponent to power law for DOS

Evangelou and Katsanos
J. Phys. A: Math. Gen. 36 3237
(2003)

Effective Support Size N^*



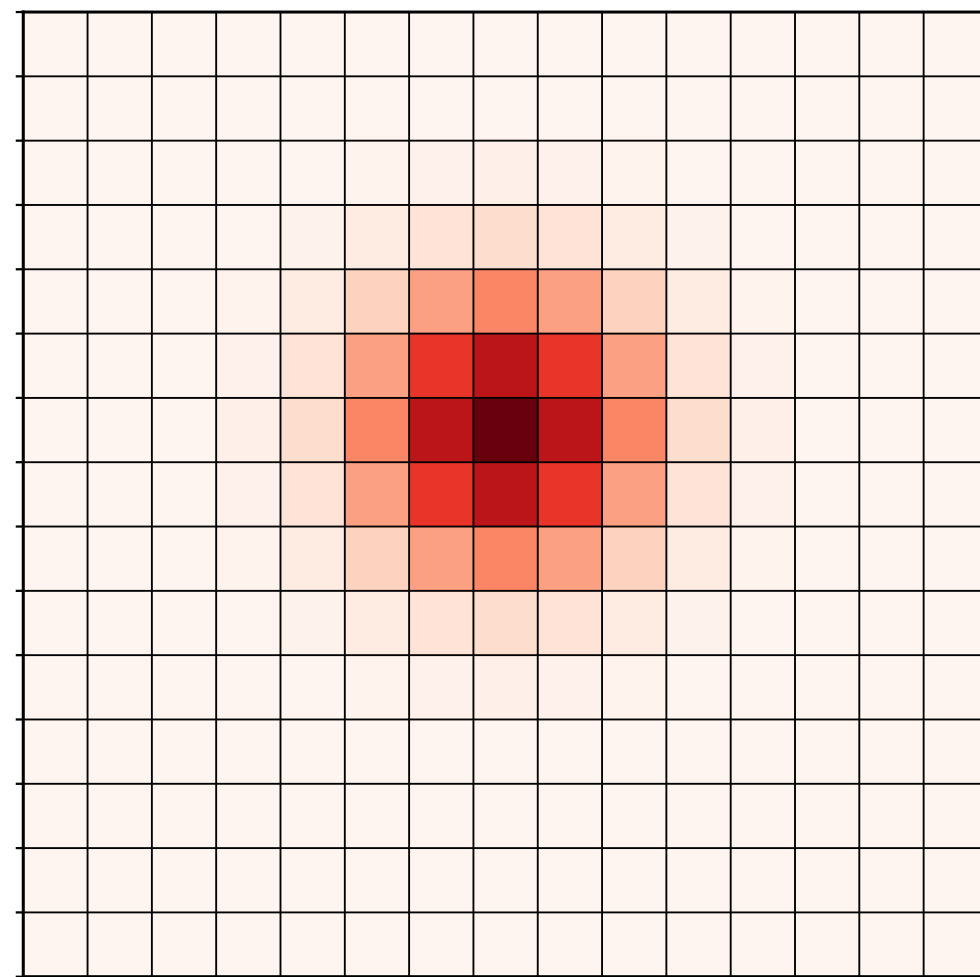
In Ivan's talk we looked at

$$c_i = N |\psi_i|^2 \quad N^* = \sum_i \min(c_i, 1)$$

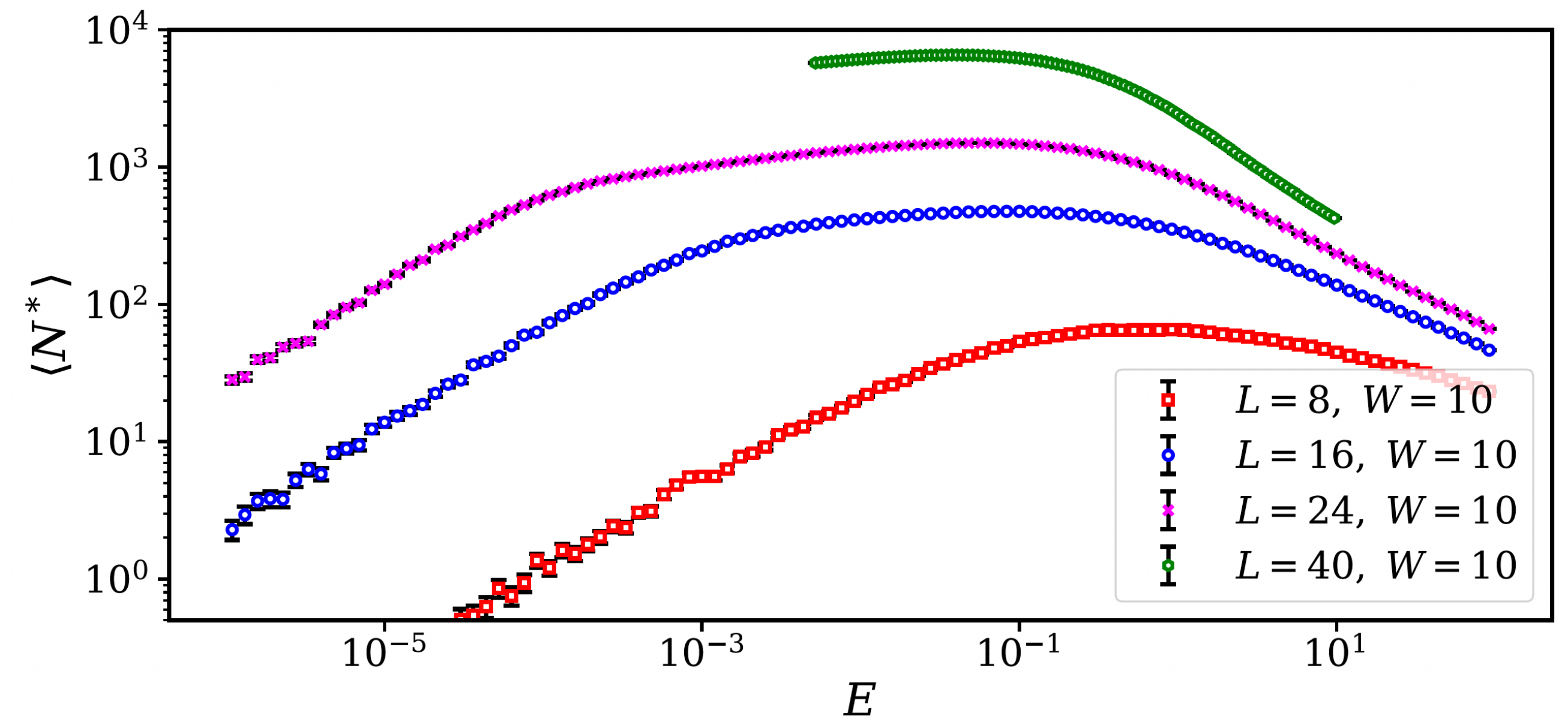
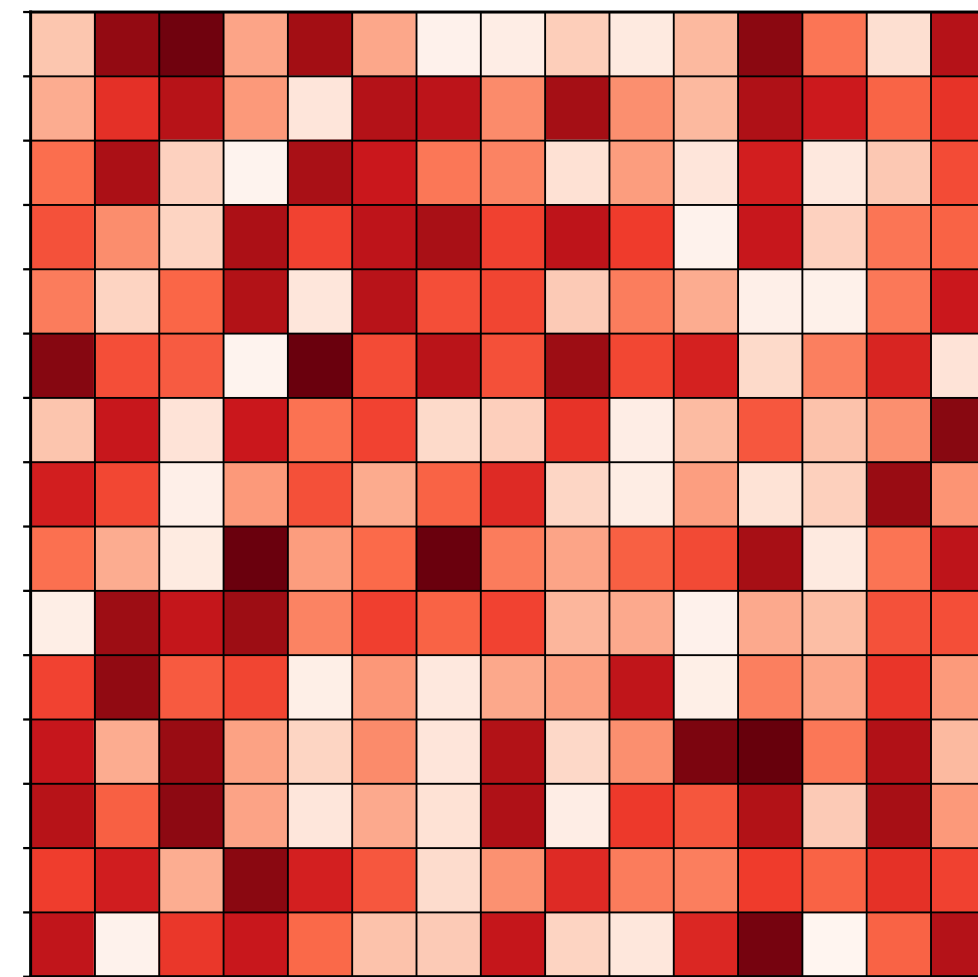
Effective number of lattice sites significantly occupied by an eigenstate

From N^* we extracted infrared dimension by finite size scaling $N^* \sim L^{d_{IR}}$

Localized state



Extended state



*Can we extract more information than a number?
Spectral decomposition (discussed next)*

Spectral Decomposition



Instead of one number (N^*), we ask:

How many of the largest-amplitude sites are needed to collect a fraction q of the eigenstate probability?

For a normalised eigenstate,

$$p_i = |\psi_i|^2, \quad \sum_i p_i = 1$$

Sort the probabilities in descending order,

$$p_{(1)} \geq p_{(2)} \geq \cdots \geq p_{(N)}$$

and define the cumulative probability

$$Q(\nu) = \sum_{i=1}^{\nu} p_{(i)}$$

The quantity $\nu(q, \psi)$ is the smallest (possibly fractional) number of sites required such that

$$Q(\nu) \geq q$$

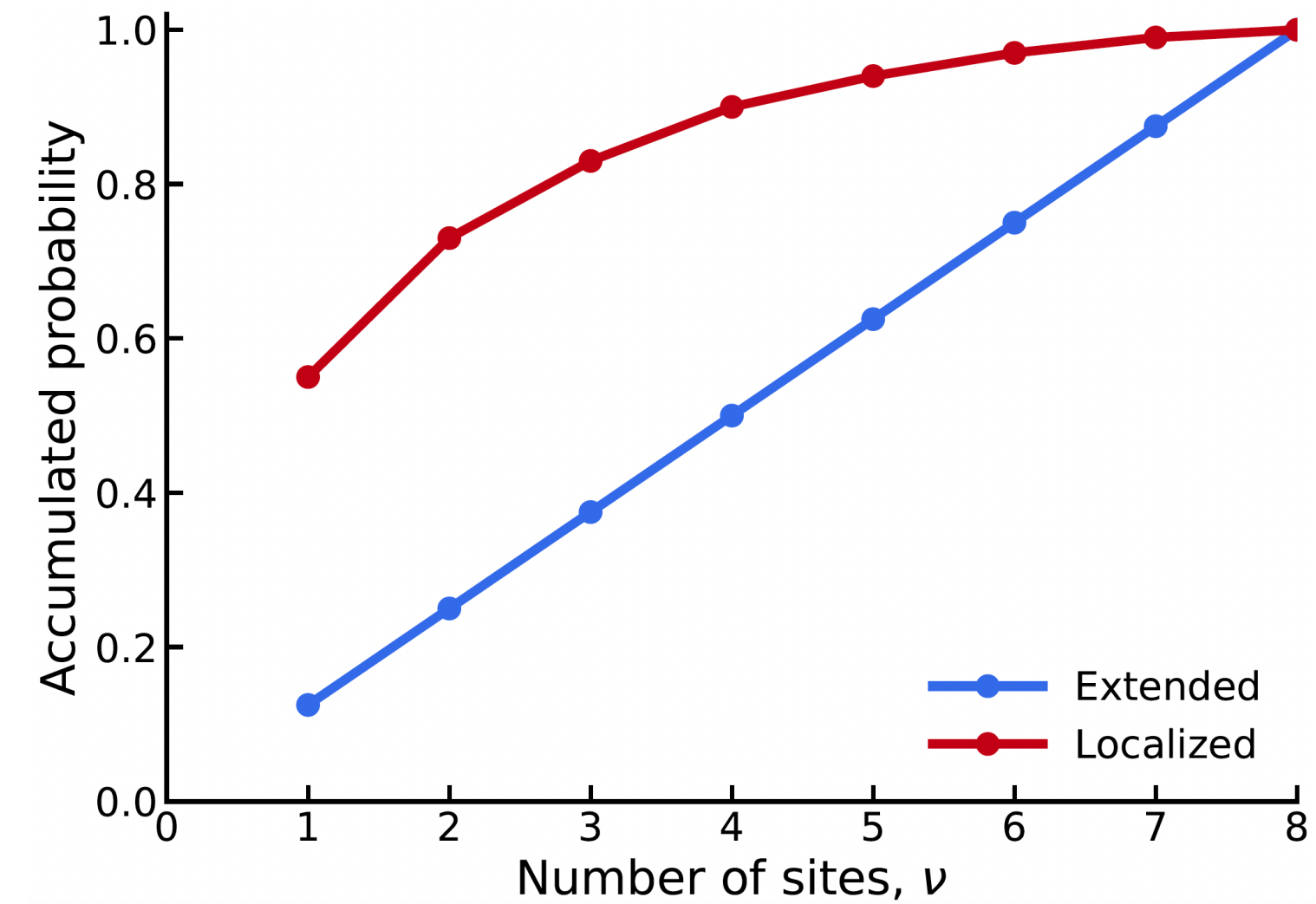
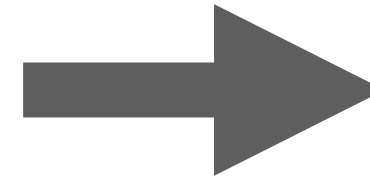
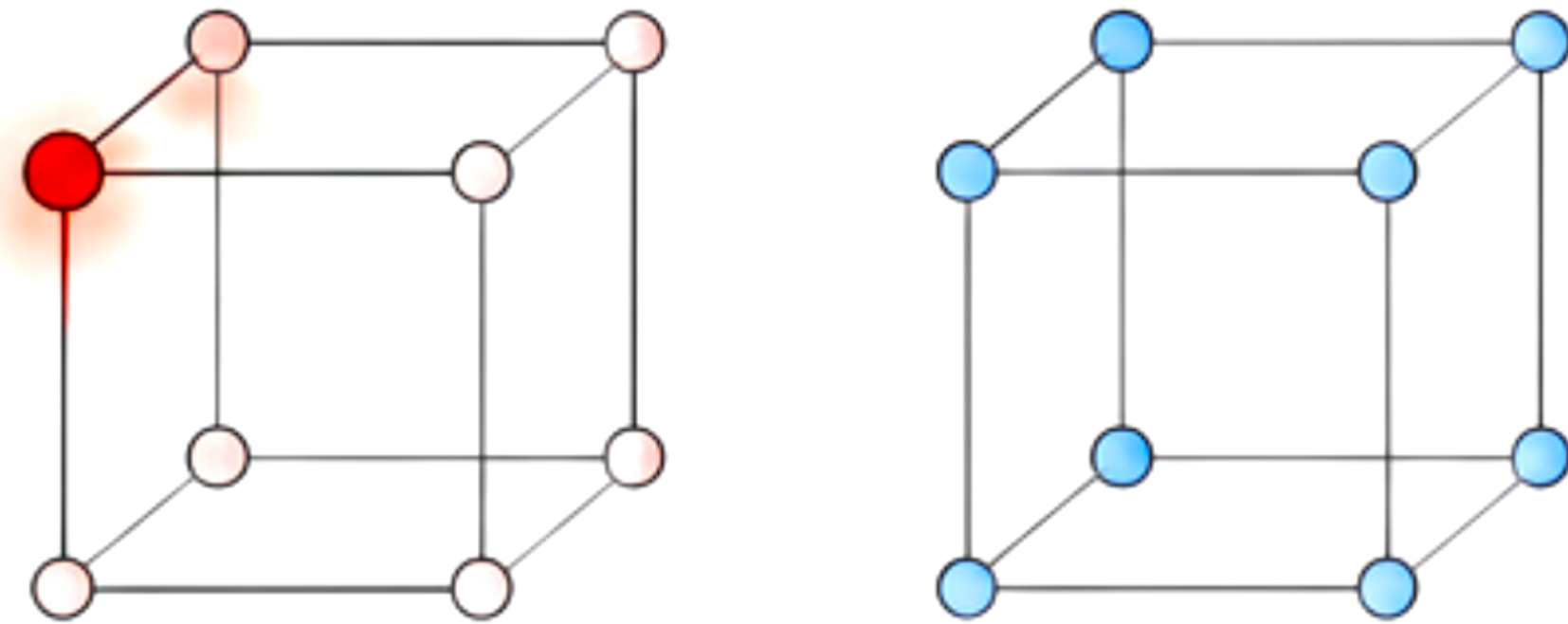
Horváth and Markoš

Entropy 25 (2023) 11, 1557

Spectral Decomposition



A toy 2^3 lattice



$$\Delta\nu(q) = \nu(q + \Delta q) - \nu(q),$$

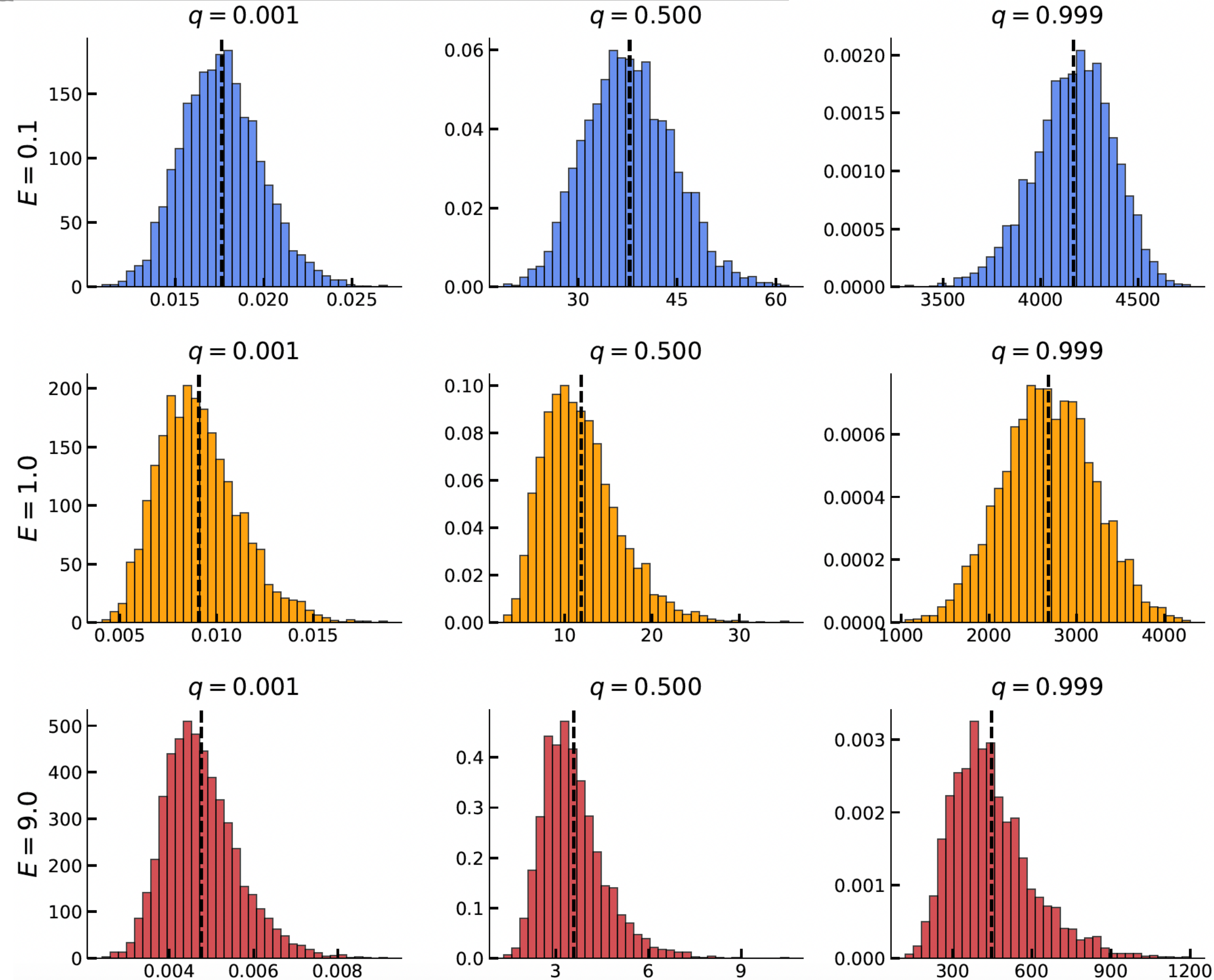
$$\Delta\nu(q, L) = A(q) L^{d(q)} e^{-c(q)/L},$$

Fitted using various lattice sizes

Counting ' ν '



Let us look at density of ν
against q and E



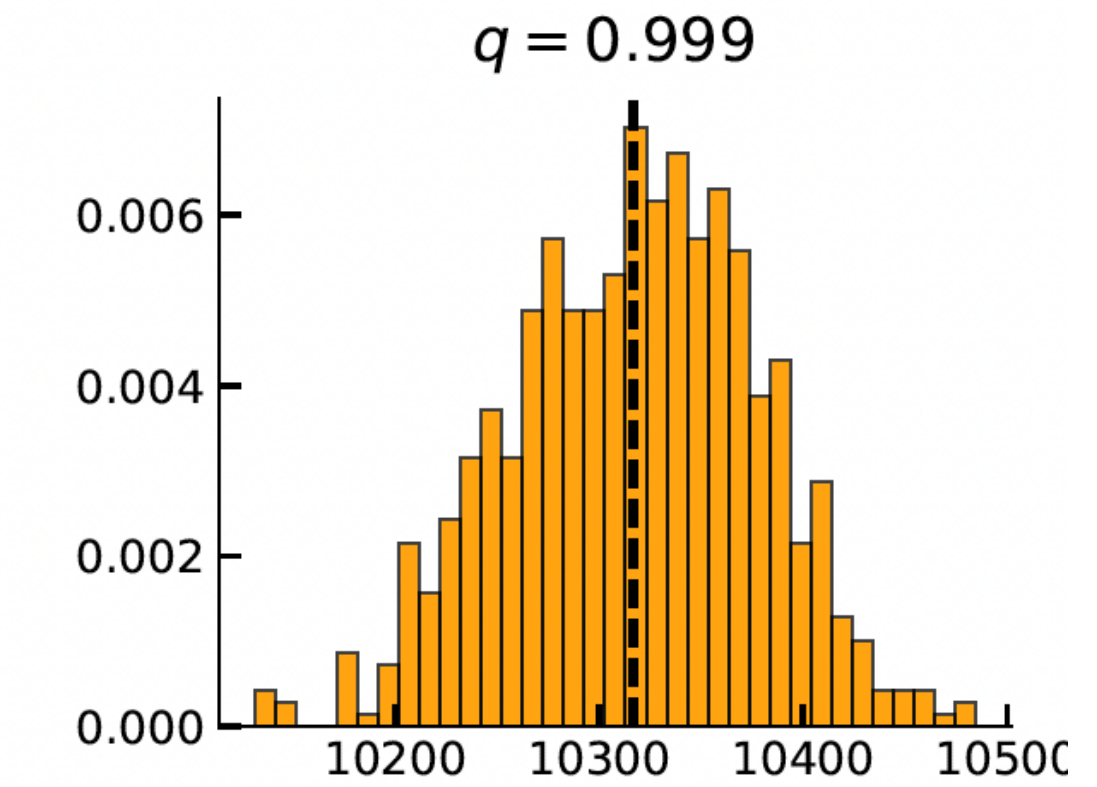
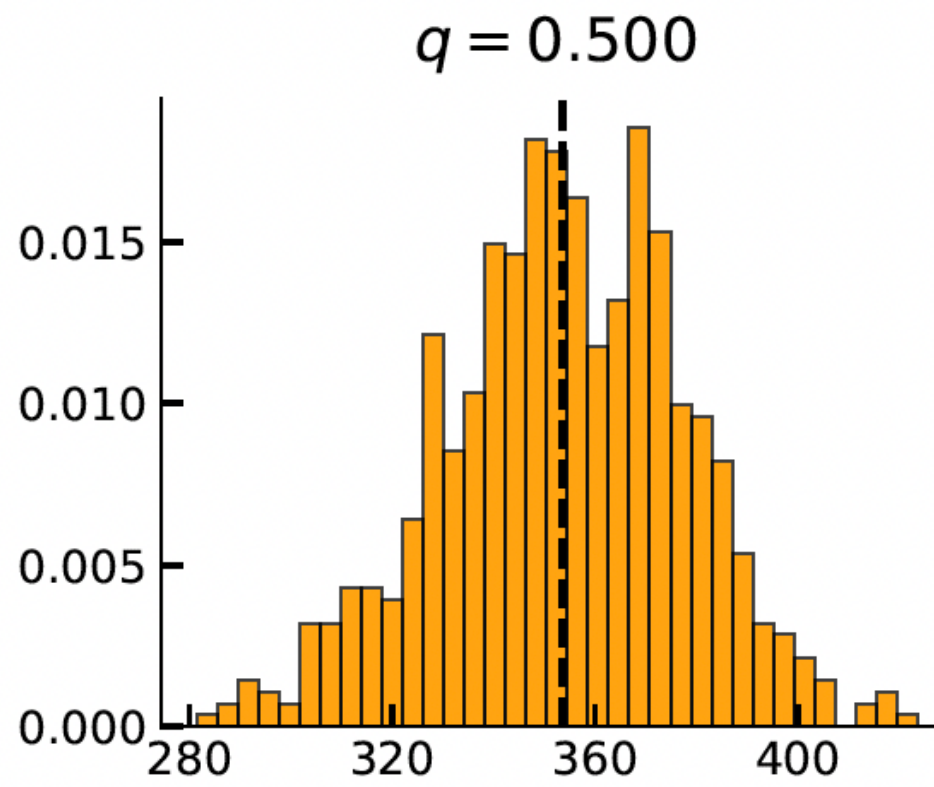
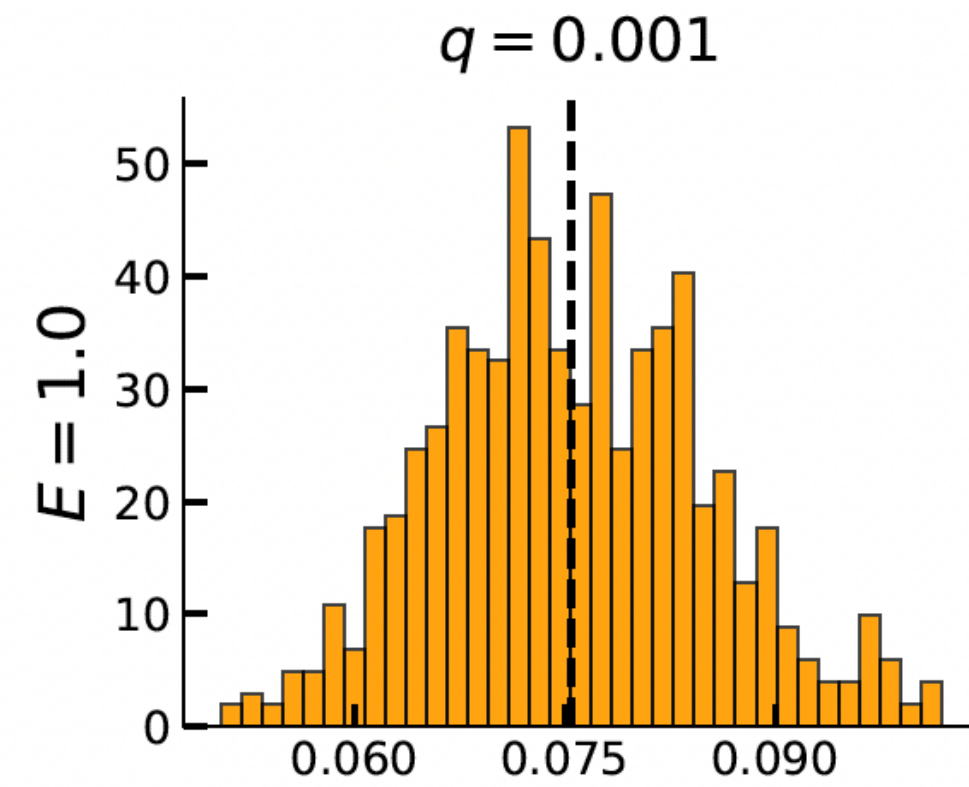
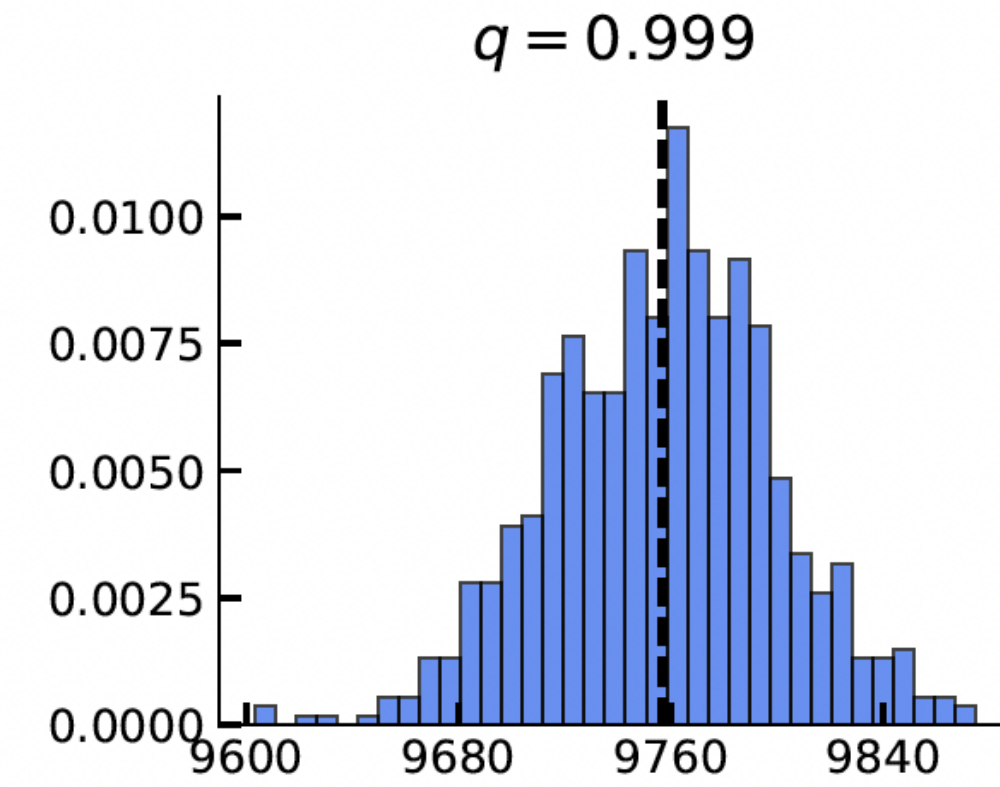
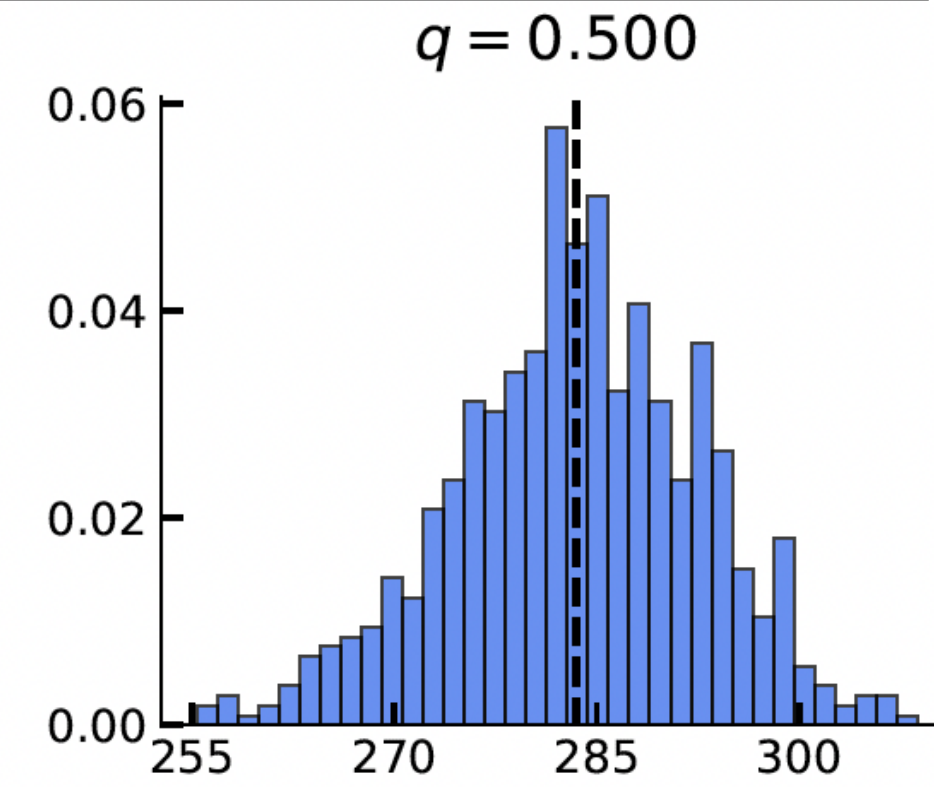
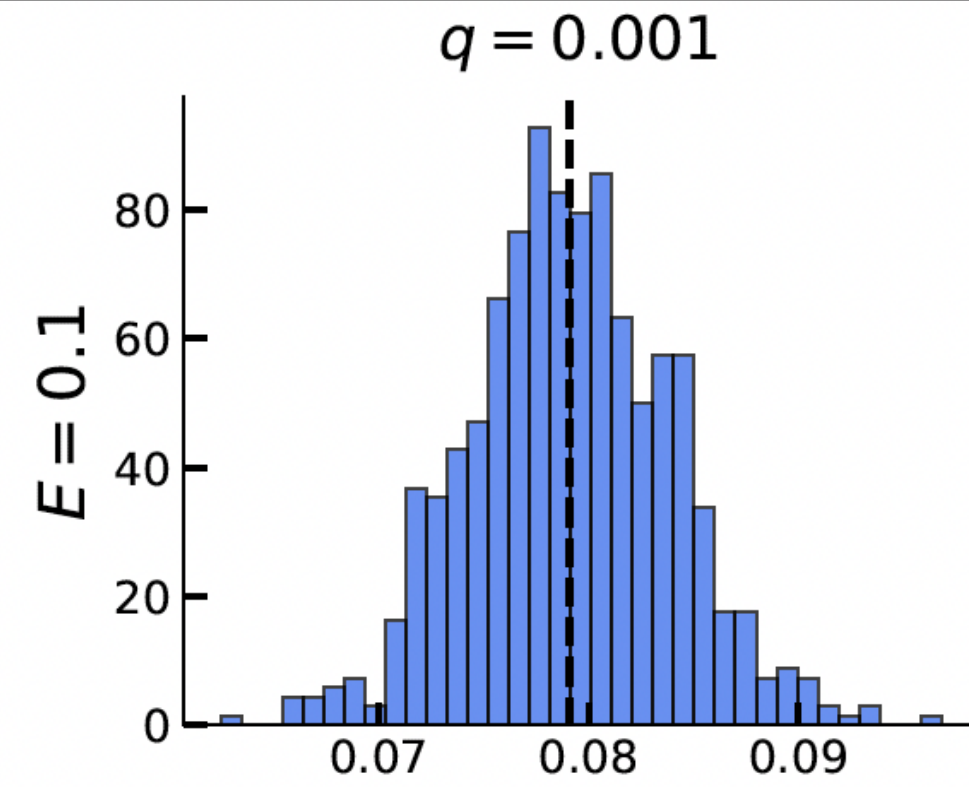
For $V = 24^3$, $W = 10$

Counting ' ν '



For $V = 24^3$, $W = 5$

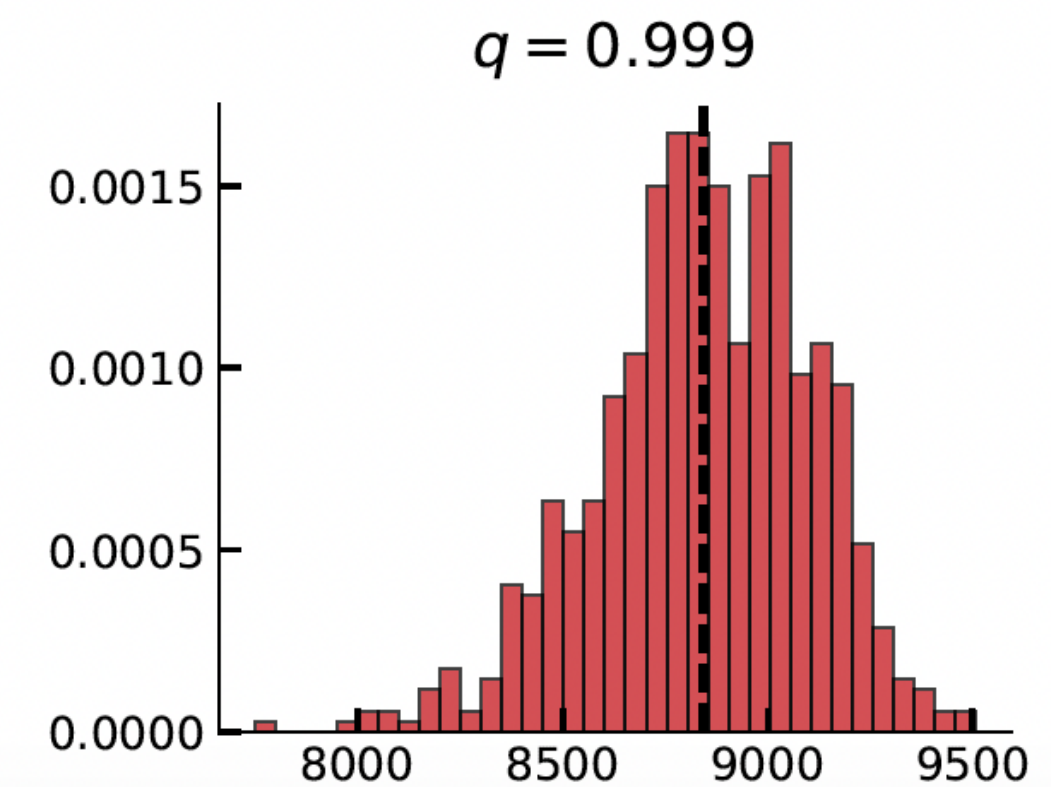
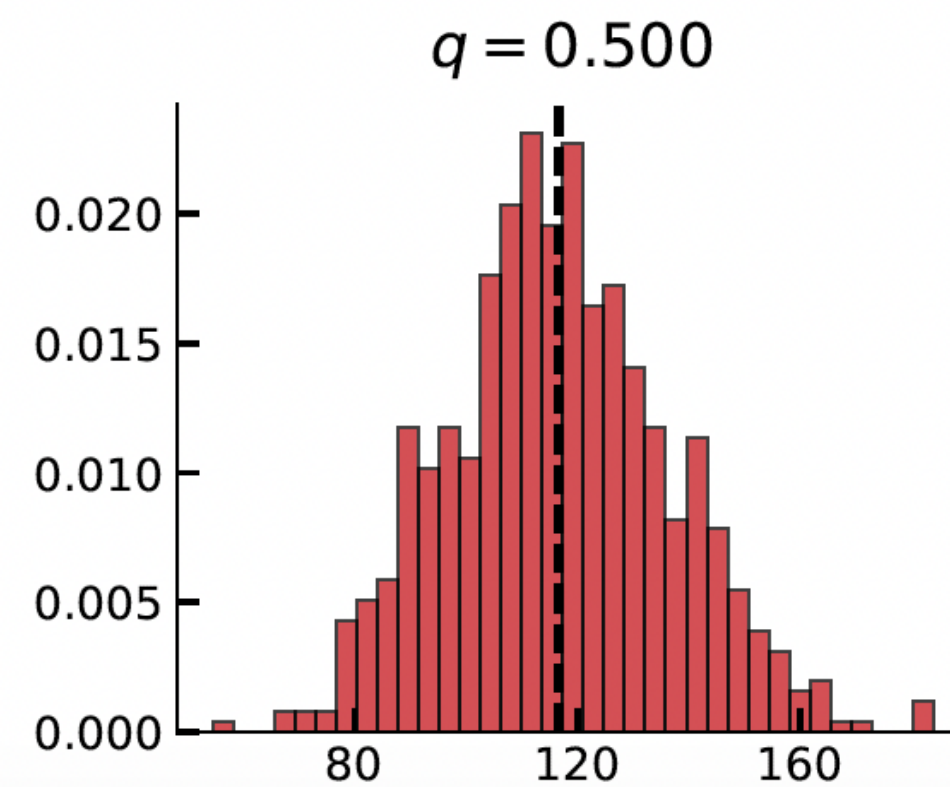
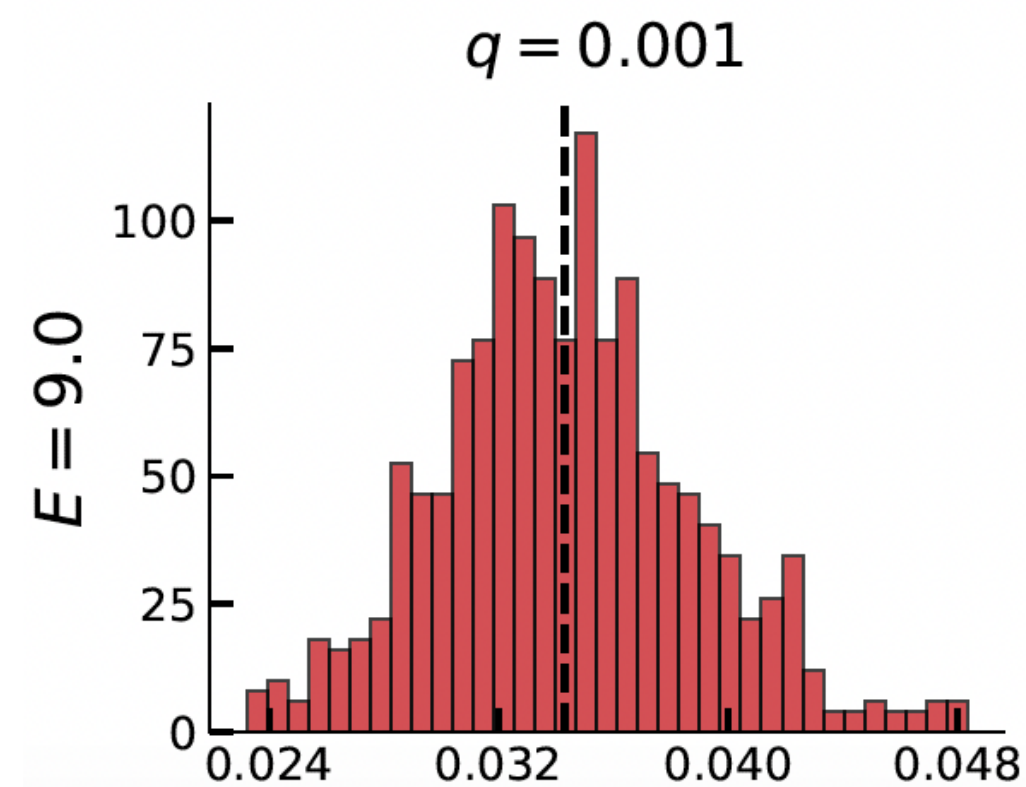
Things changes
significantly



Limited statistics

+

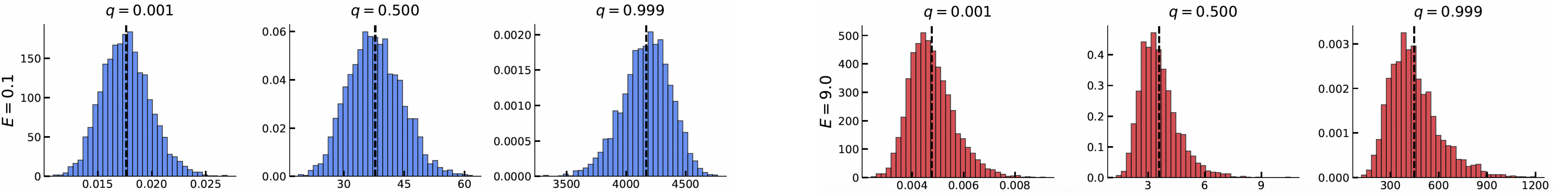
Finite Volume



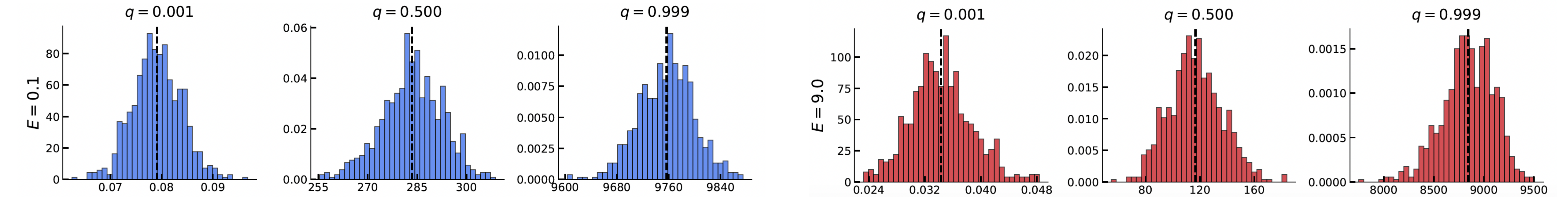
Counting ' ν '



$W = 10$



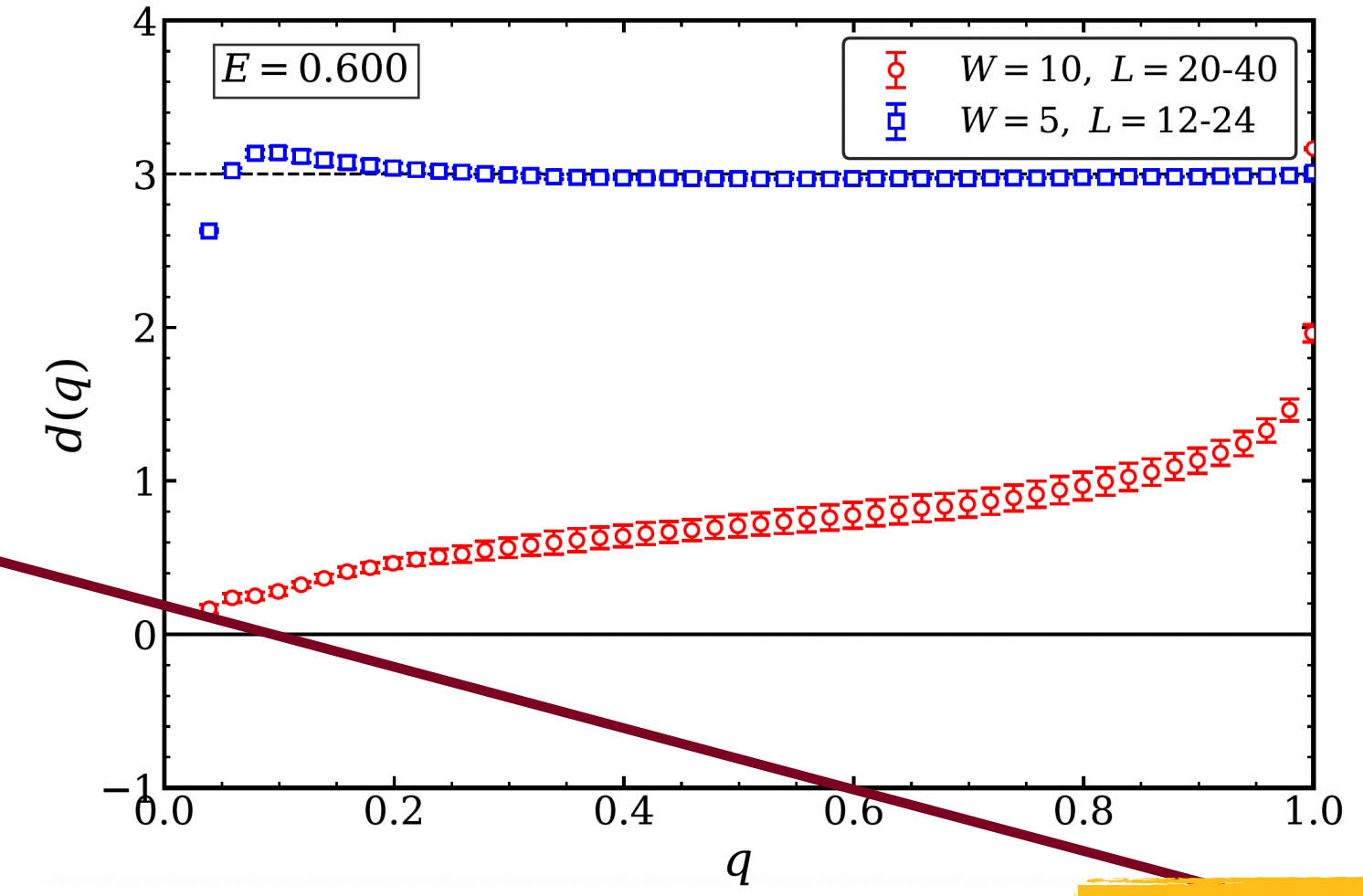
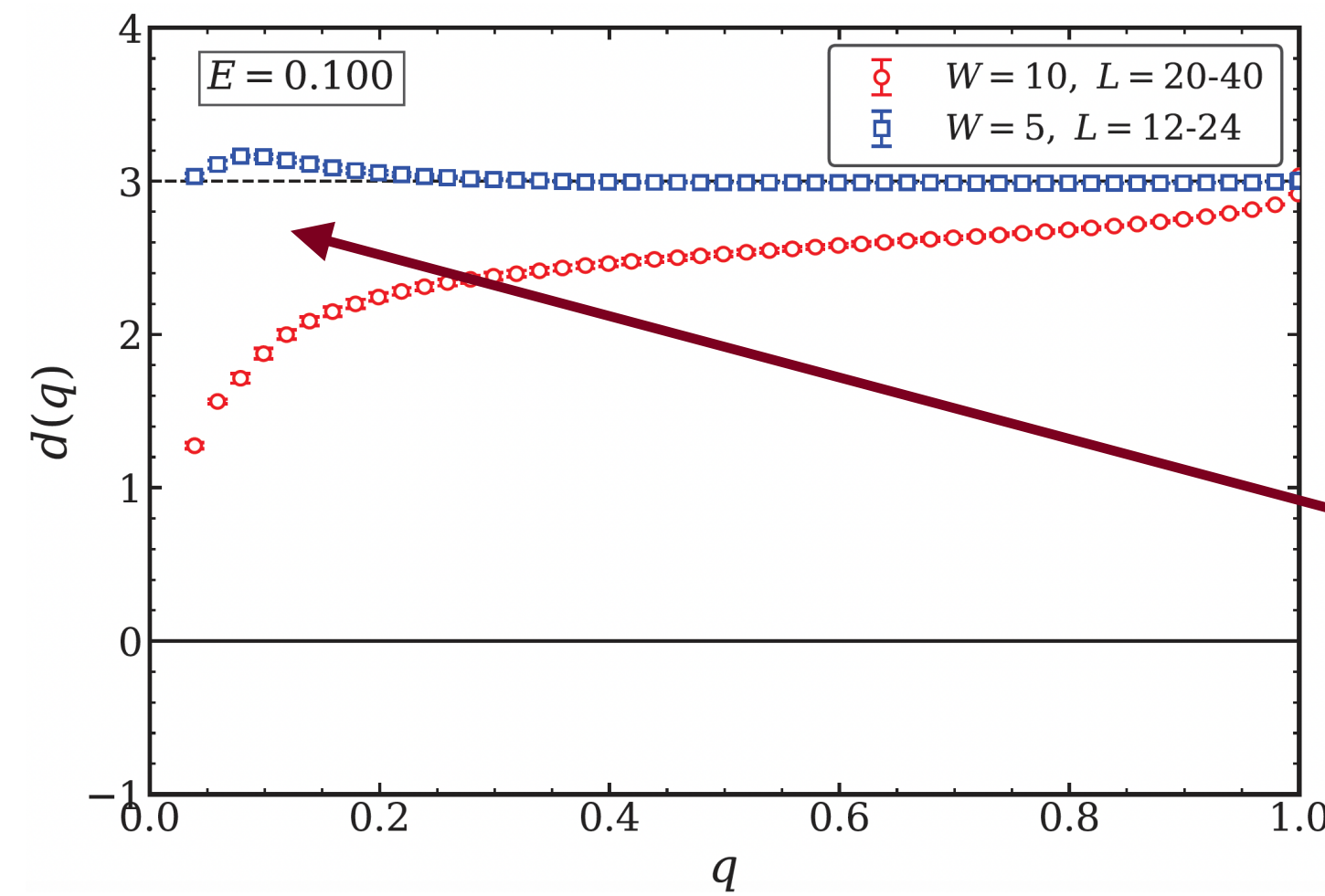
$W = 5$



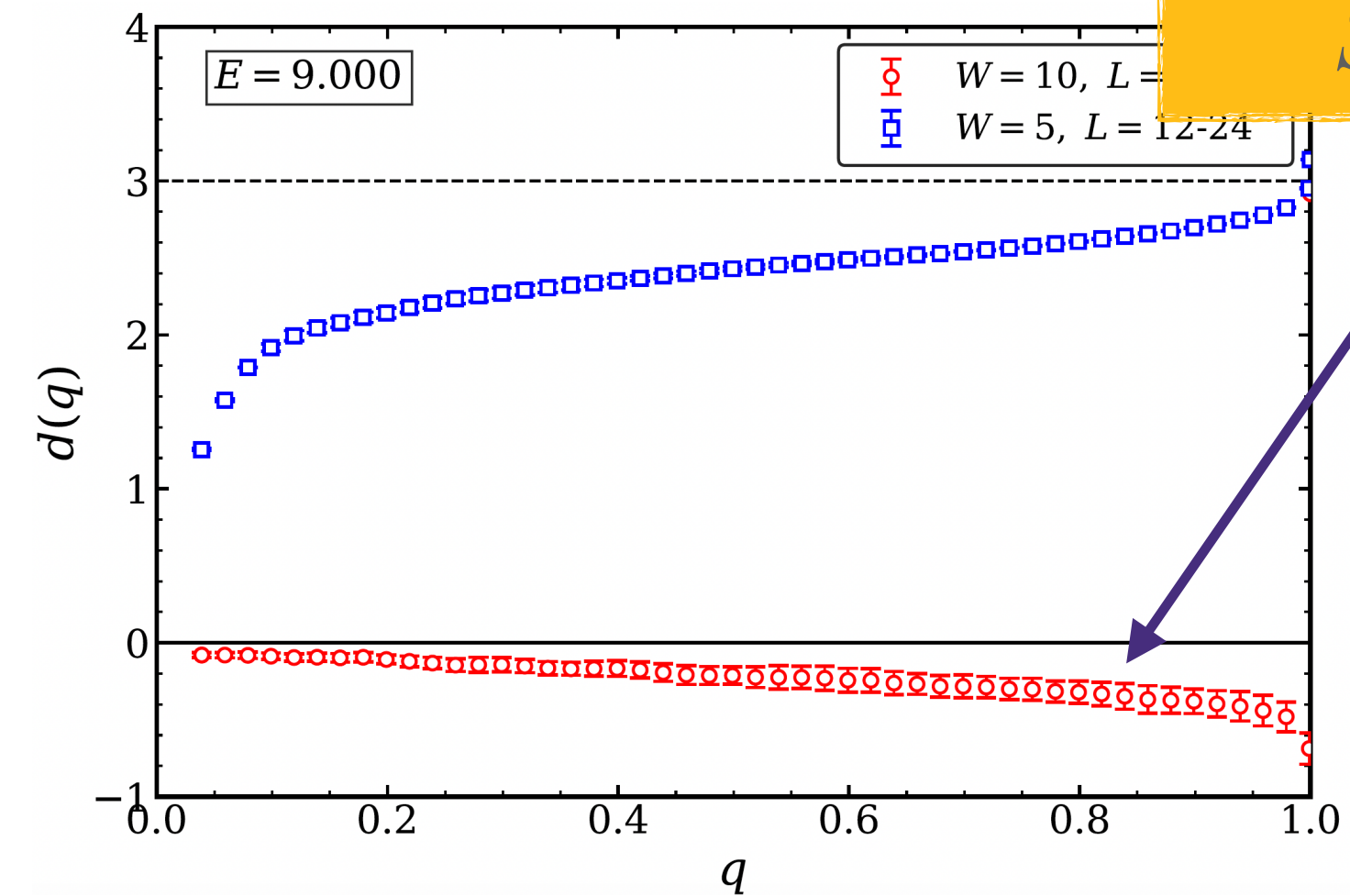
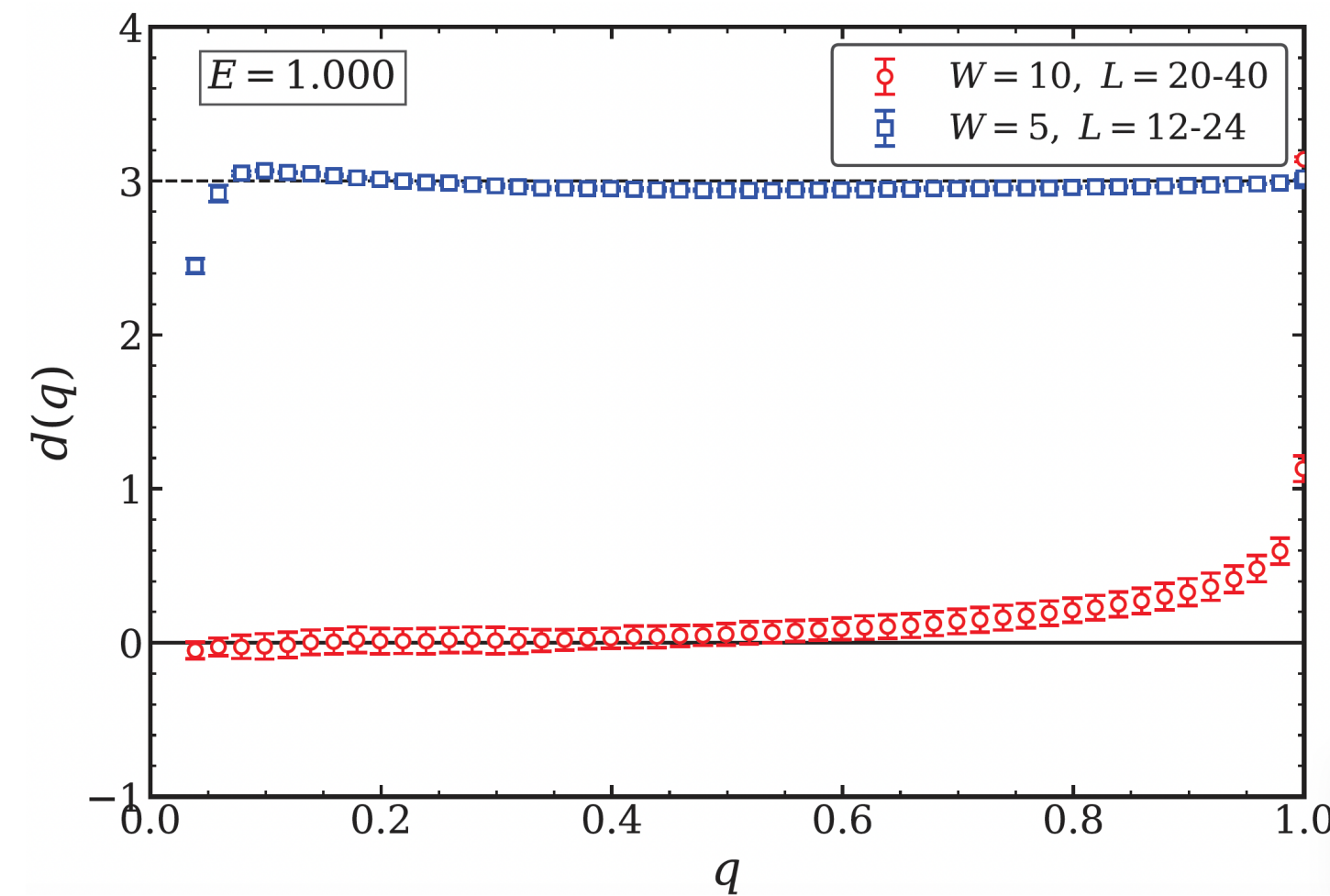
$d(q)$



► Extracting the effective dimension $d(q)$ from the scaling behaviour of $\nu(q)$



$$\Delta\nu(q, L) = A(q) L^{d(q)} e^{-c(q)/L},$$

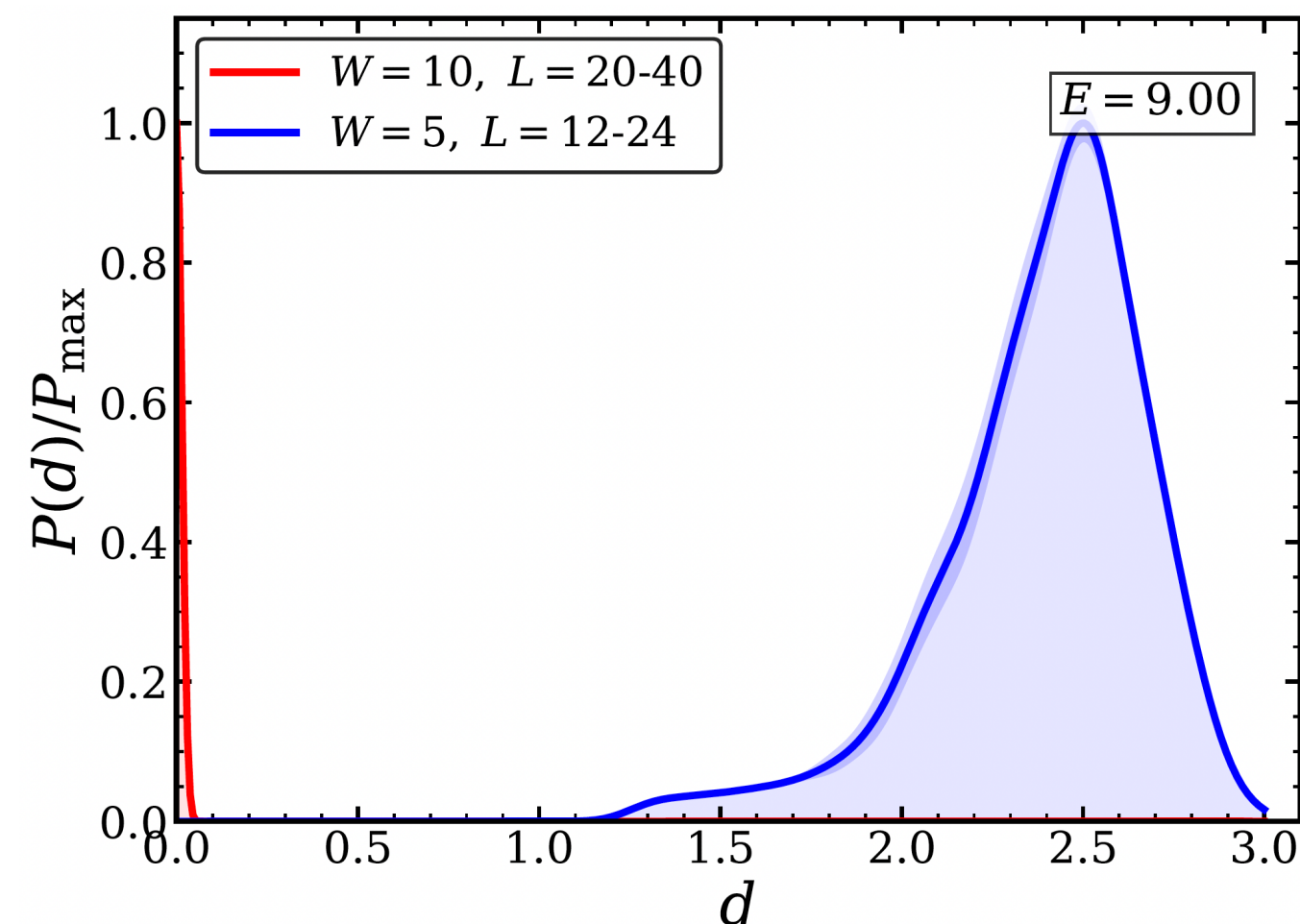
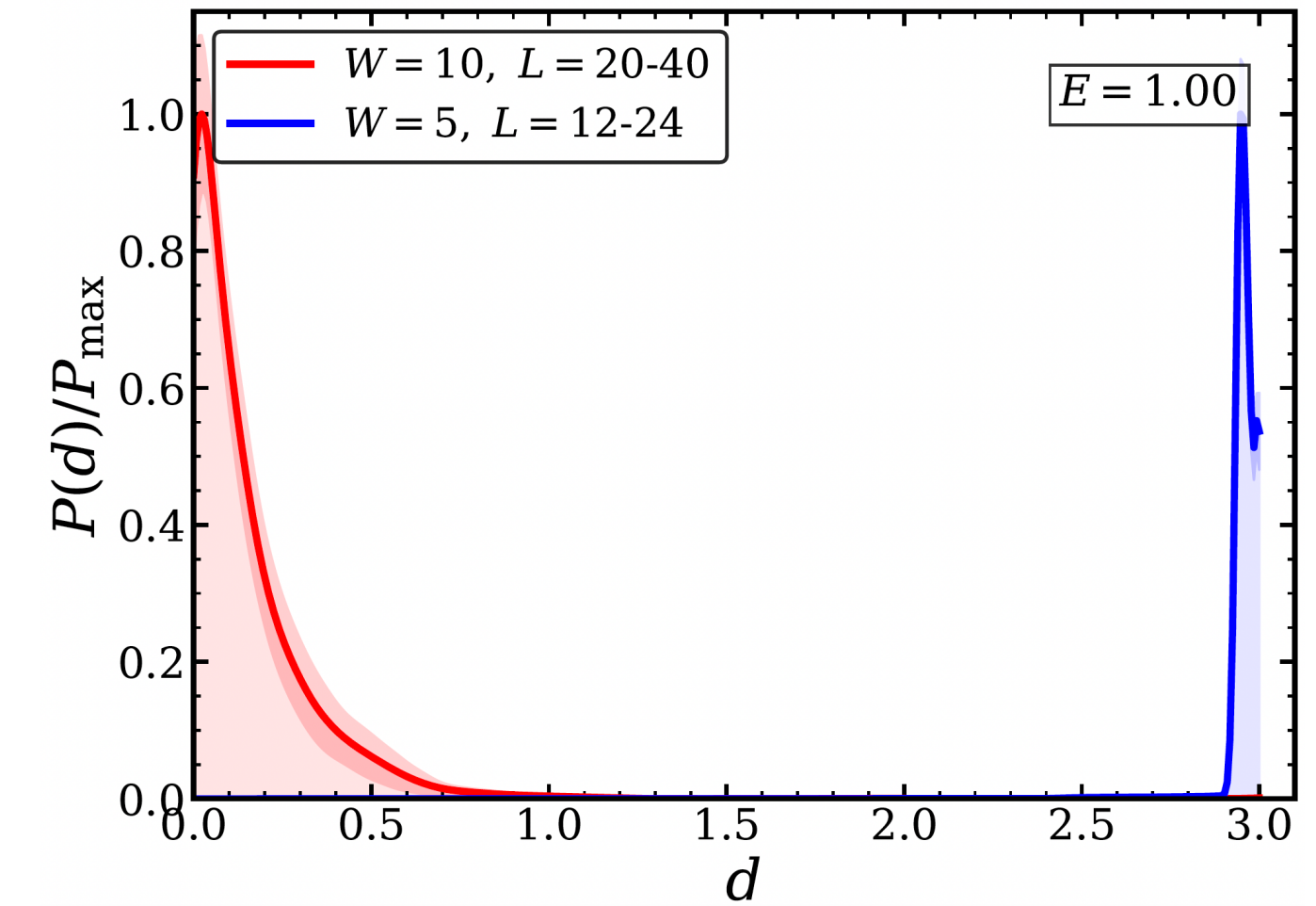
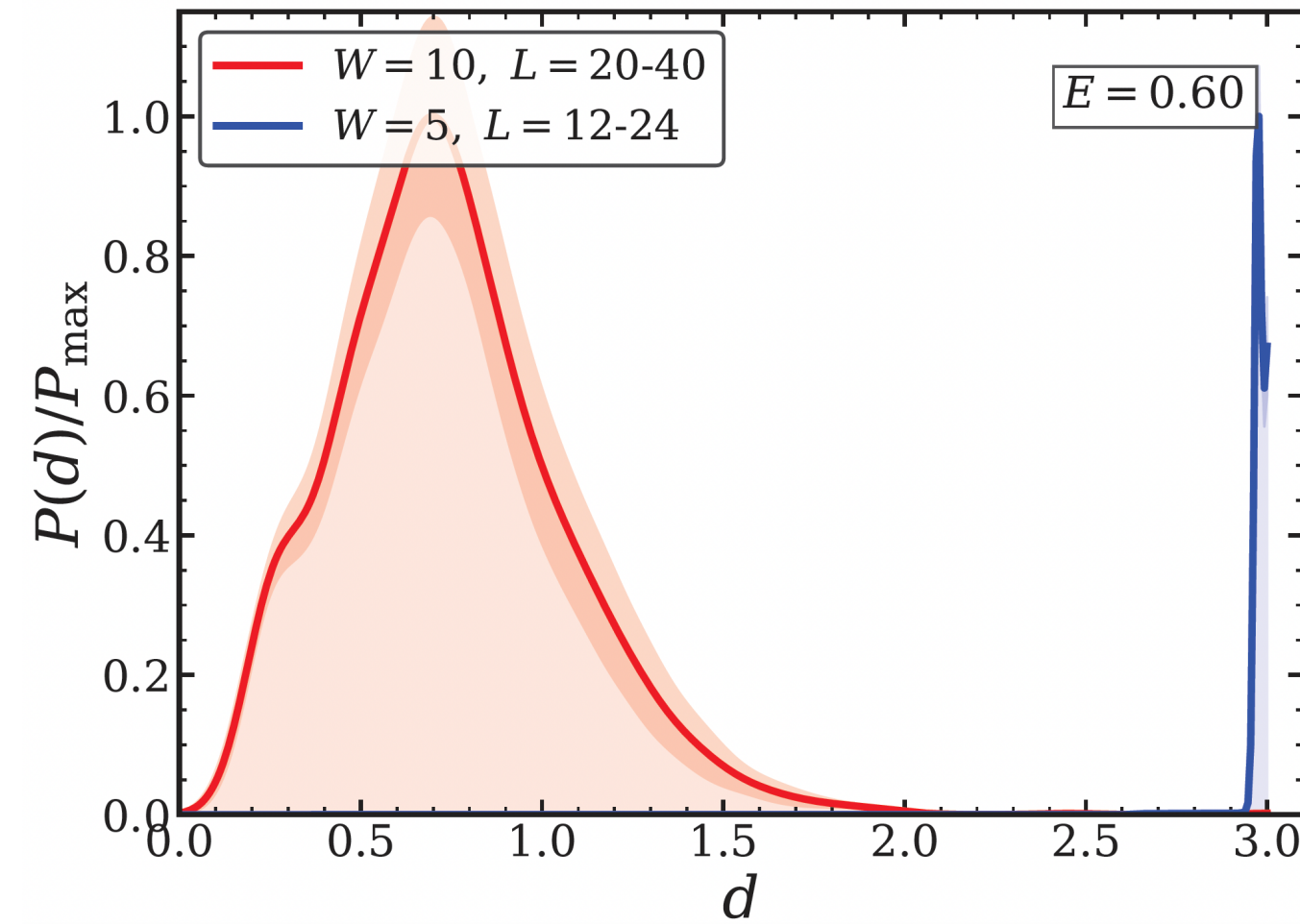
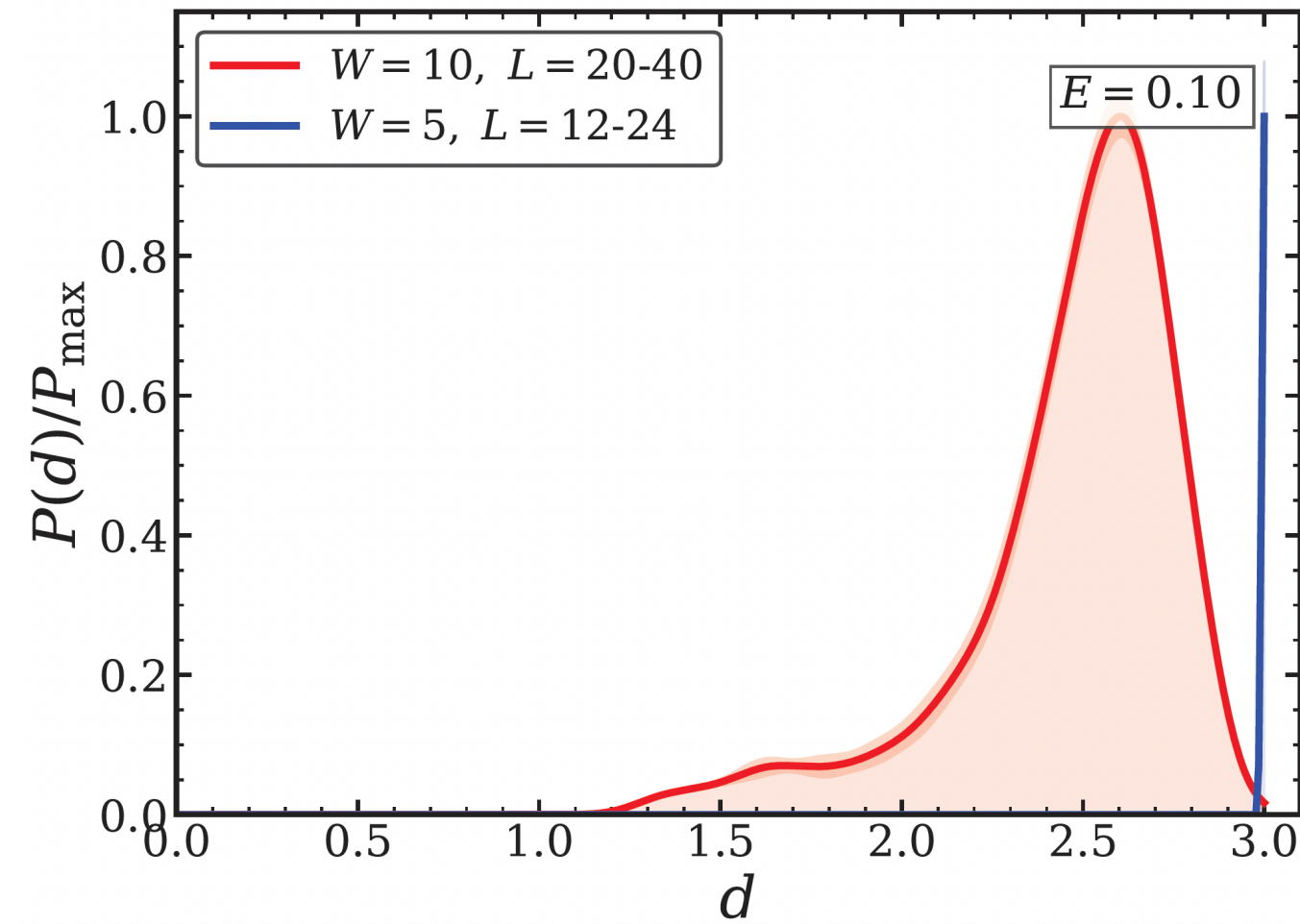


*More data...
Sensitive fits...*

$$P(d)$$



► Constructing the probability distribution $P(d)$ using Kernel Density Estimation (KDE)

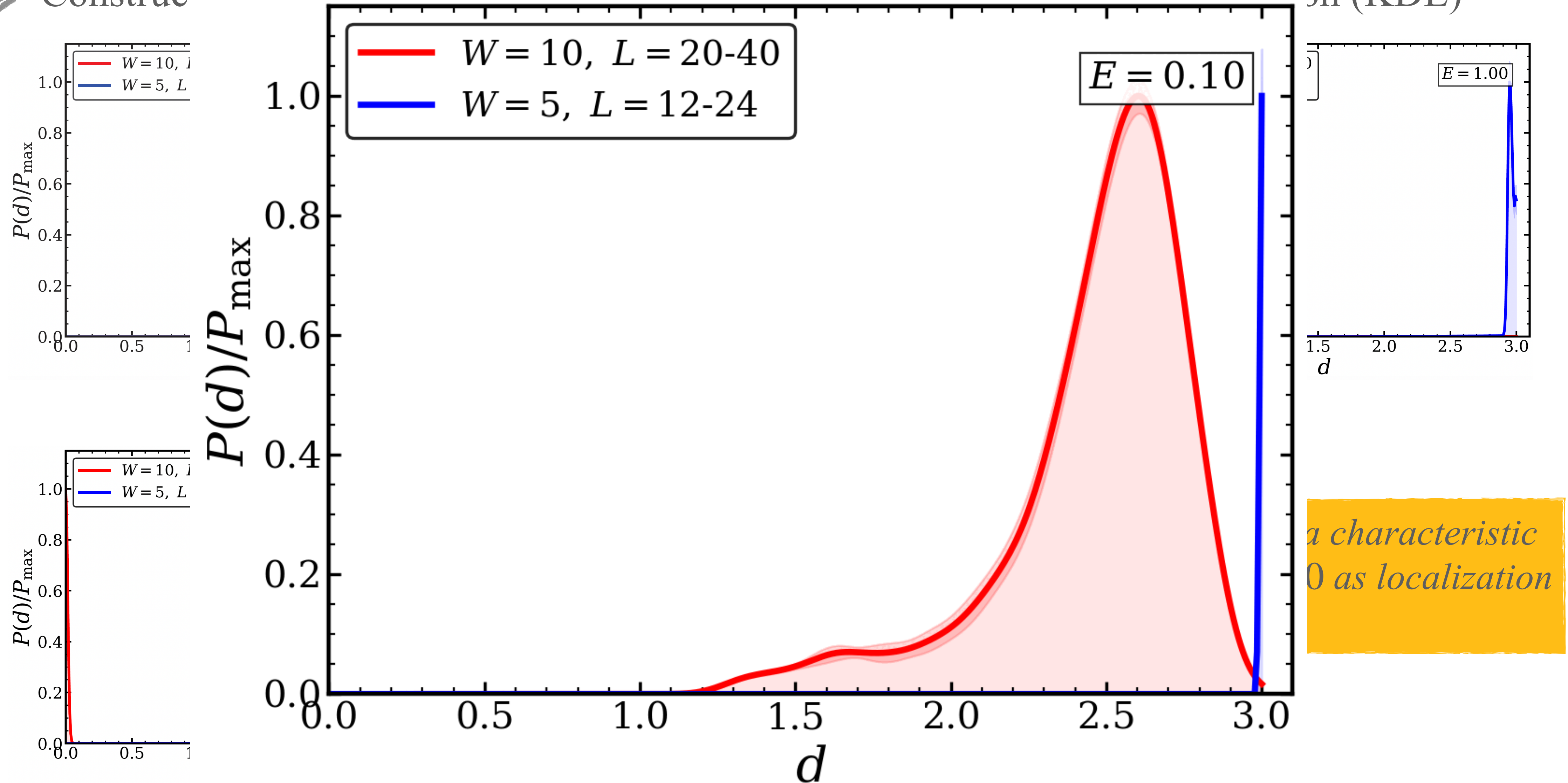


KDE summarizes the full $d(q)$ spectrum into a characteristic dimension. The peak shifts from $d \sim 3$ to $d \sim 0$ as localization develops

$$P(d)$$



► Constructing the probability distribution $P(d)$ using Kernel Density Estimation (KDE)

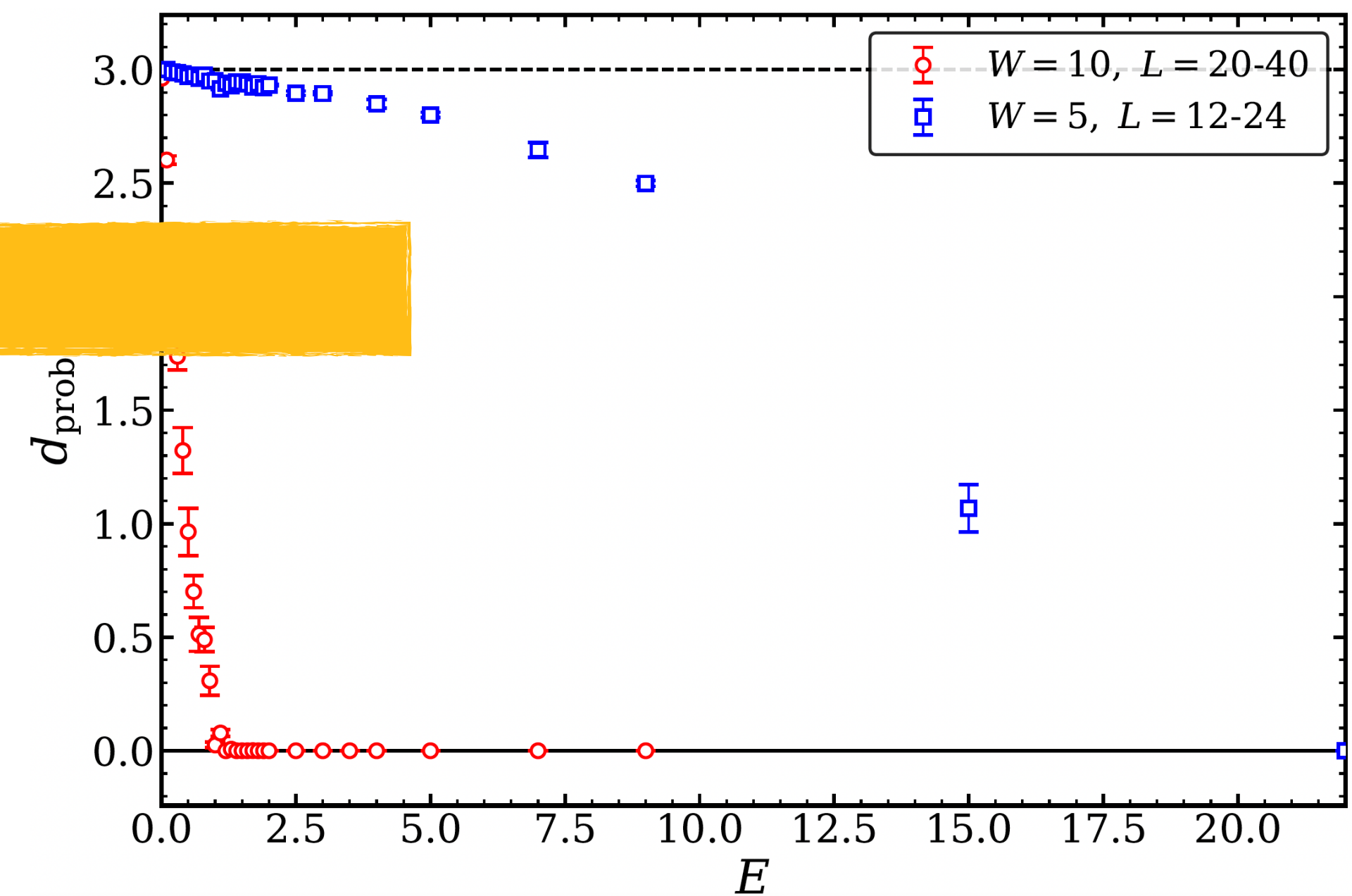
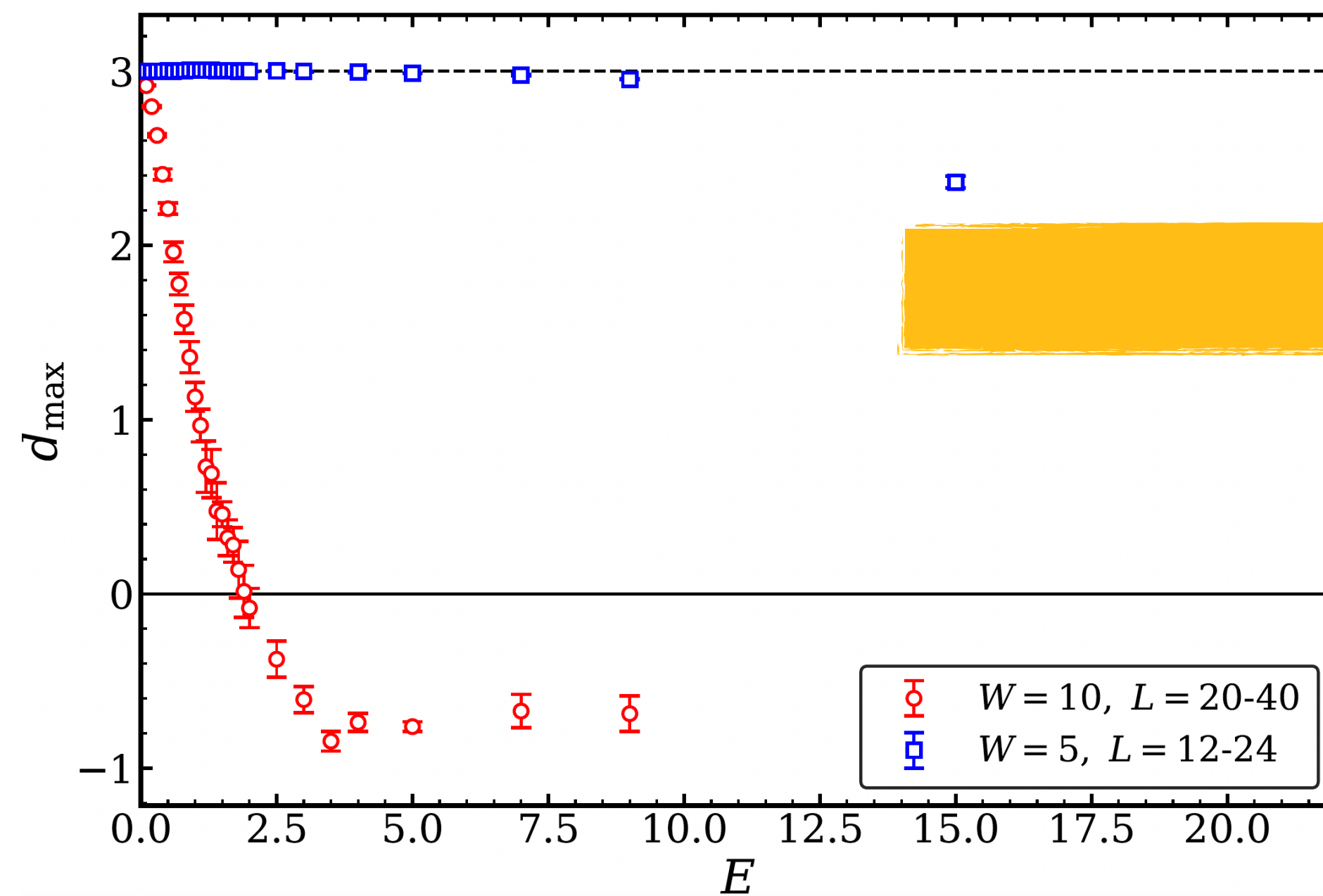


Maximal and Probable Dimension



► $d_{max}(q = 0.999)$: the dimension obtained after capturing 99.9% of the eigenstate probability, characterizing the geometry of the full support.

► d_{prob} : the most probable geometric dimension, extracted from the peak of the KDE distribution $P(d)$



Still exploring

► Weak disorder: $d_{max}, d_{prob} \sim 3$ over most energies

► Strong disorder: both dimensions decrease with energy, revealing the localization transition

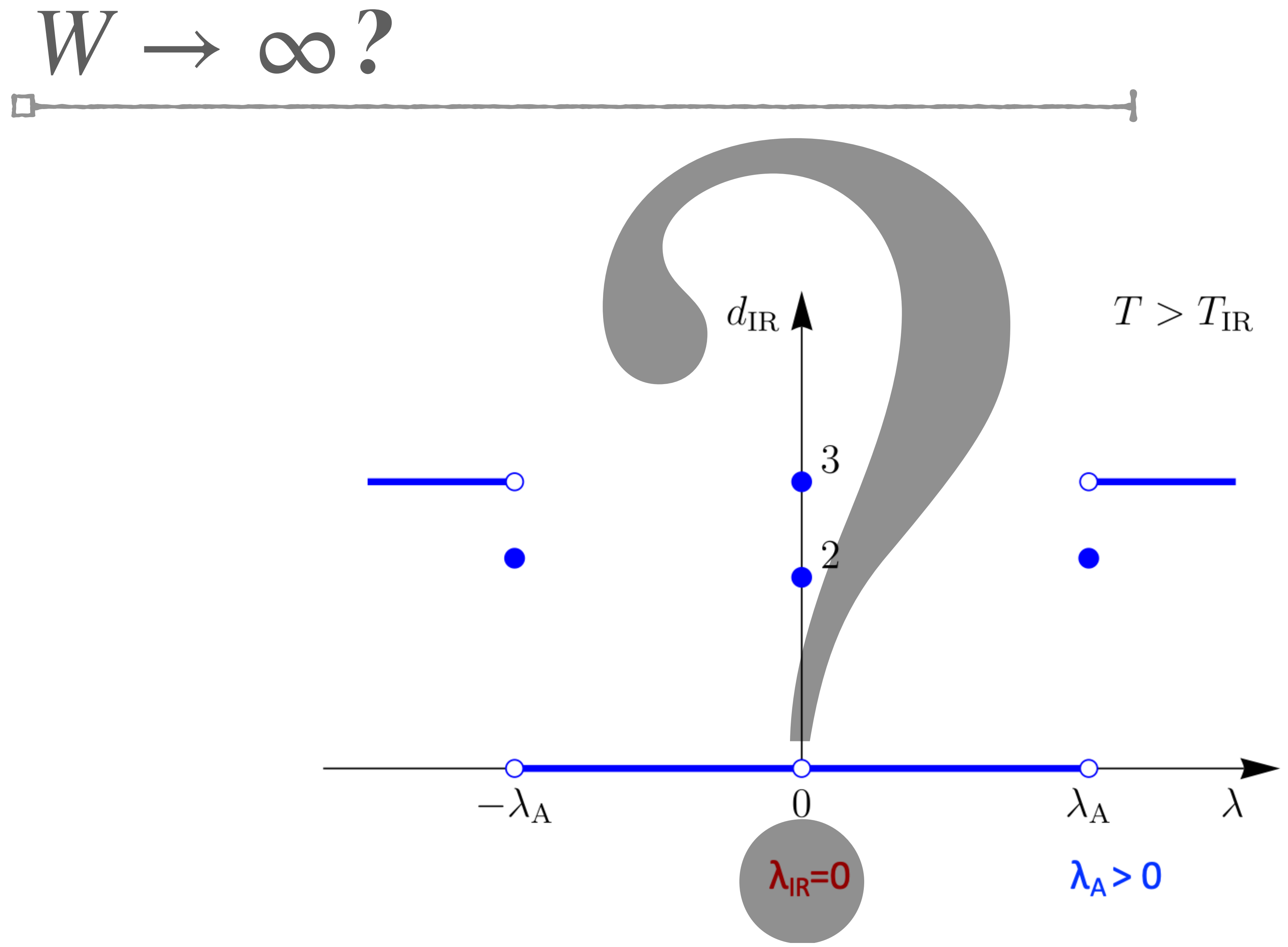


Figure from Ivan

Summary



- ☒ We studied chiral Anderson models: randomly disordered systems with chiral (sublattice) symmetry
- ☒ Numerical calculations were performed on finite lattices of volume, $V = L^3$, with $L = 12 \rightarrow 40$
- ☒ We explored how the effective spatial dimension of eigenstates depends on energy and disorder (hopping) strength
- ☒ The effective dimension provides a new way to characterize the geometry of eigenstates
- ☐ Larger volumes? Different types of disorders?
- ☐ Better way of understanding dimensions?
- ★ And more as mentioned in Ivan's talk

THANK YOU

Navdeep Singh Dhindsa

02/08/26



tifr.link/scicomm