# Deconfinement phase transition in bosonic BMN model at general coupling


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## 1. Introduction

- Lower-dimensional models of matrices can capture the dynamics of string/M-theory in an appropriate limit of the parameters. Banks, Fischler, Susskind, and Shenker (BFSS) provided the equivalence between dimensionally reduced ten-dimensional $\mathcal{N} = 1$ super-Yang–Mills (SYM) with gauge group SU($N$) and M-theory in the light-cone gauge, in the large-$N$ planar limit.
- Berenstein, Maldacena, and Nastase extended this model by introducing a supersymmetry-preserving one-parameter deformation, known as the BMN matrix model, which describes a certain limit of Type II string theory on a pp-wave background, rather than the flat spacetime relevant for the BFSS model.
- At large $N$ and finite temperature, there are often phase transitions between different quantum black hole solutions, which are dual to confinement transitions in the field theory. In this regard, the D0 brane matrix model is an exception, with only a single deconfined phase at all temperatures in the planar limit.
- Our goal is to explore the functional form of the dependence of the bosonic BMN critical temperature $\widehat{T}_c$ on the deformation parameter $\widehat{\mu}$. We analyze twelve different values of $\widehat{\mu}$ between $\widehat{\mu} \to 0$ and $\widehat{\mu} \to \infty$ limits.

## 2. Our setup

- No fermions.
- Well-defined phase transition even without deformation of BFSS model.
- Easier to study larger lattice sizes and large-$N$ by decoupling the fermions.
- Phase transition and its order are monitored using the variance of the Polyakov loop.
- Scalar mass terms break SO(9) global symmetry of the BFSS model down to SO(3) $\times$ SO(6).
- Number of lattice sites, $N_\tau$ is fixed as 24 for all runs.

## 3. Lattice observables

Lattice action is obtained by discretizing the bosonic BMN model on a lattice with $N_\tau$ sites

$$S_{\text{lat}} = \frac{N}{4\lambda_{\text{lat}}} \sum_{n=0}^{N_\tau - 1} \text{Tr} \left[ -(\mathcal{D}_+ X_i)^2 - \frac{1}{2} \sum_{i<j} [X_i, X_j]^2 - \left(\frac{\mu_{\text{lat}}}{3} X_I\right)^2 - \left(\frac{\mu_{\text{lat}}}{6} X_A\right)^2 + \frac{\sqrt{2}\mu_{\text{lat}}}{3} \epsilon_{IJK} X_I X_J X_K \right].$$

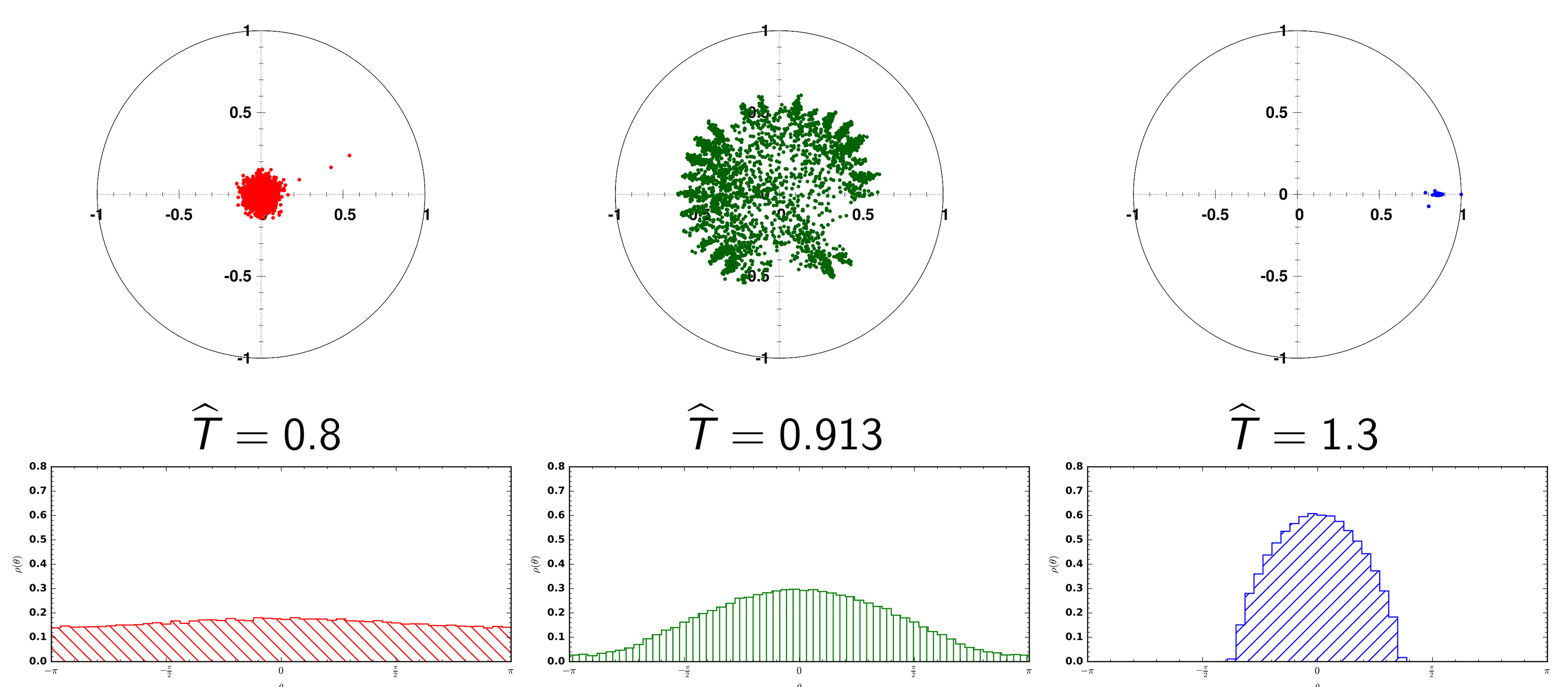
The indices runs from $i, j = 1, \cdots, 9$, $I, J, K = 1, 2, 3$, and $A = 4, \cdots, 9$. The finite-difference operator is given as $\mathcal{D}_+ X_i(n) \equiv U(n) X_i(n+1) U^\dagger(n) - X_i(n)$. Lattice action is written in dimensionless lattice parameters $\mu_{\text{lat}} \equiv a\mu$ and $\lambda_{\text{lat}} \equiv a^3 \lambda$. Following are some useful parameters:

$$\widehat{T} \equiv \frac{T}{\lambda^{1/3}} = \frac{1}{N_\tau \lambda_{\text{lat}}^{1/3}}, \qquad \widehat{\mu} \equiv \frac{\mu}{\lambda^{1/3}} = \frac{\mu_{\text{lat}}}{\lambda_{\text{lat}}^{1/3}}, \qquad \frac{\widehat{T}}{\widehat{\mu}} = \frac{T}{\mu} = \frac{1}{N_\tau \mu_{\text{lat}}}.$$

Polyakov loop magnitude $|P|$ on lattice and its susceptibility, $\chi$, which are used as order parameter to tune different phases

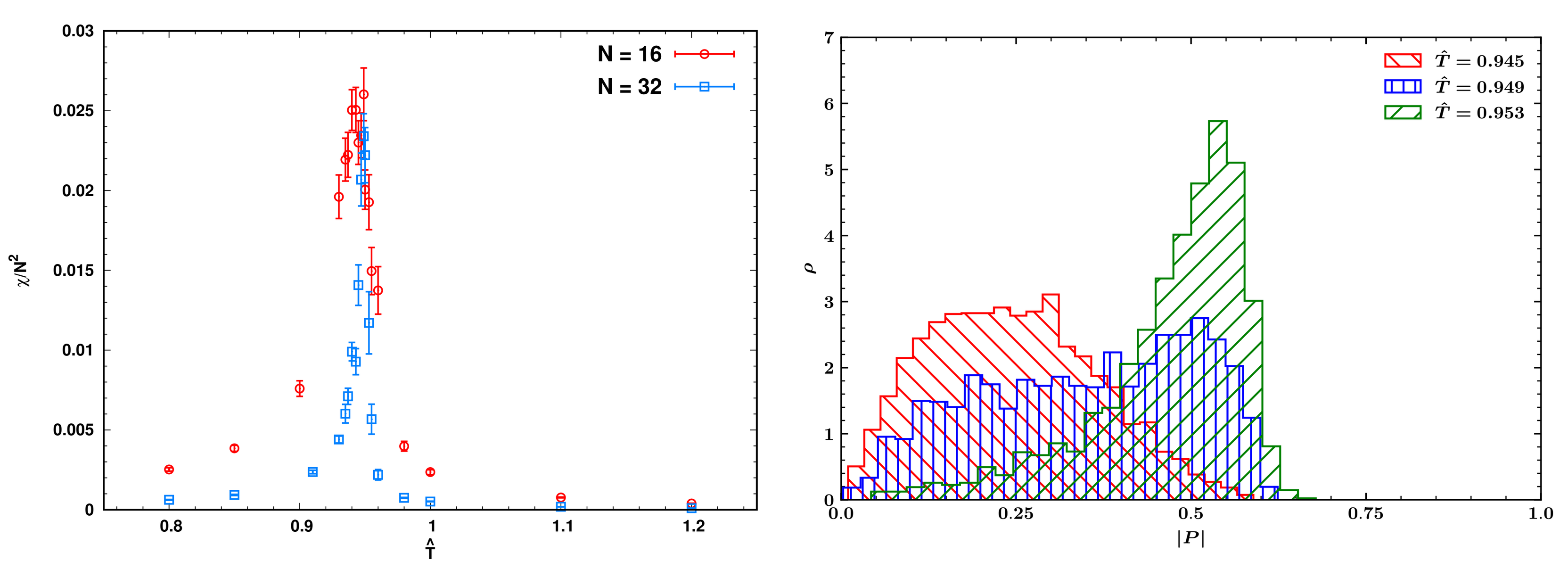
$$|P| = \left\langle \frac{1}{N} \left| \text{Tr} \left( \prod_{n=0}^{N_\tau - 1} U(n) \right) \right| \right\rangle, \qquad \chi \equiv N^2 \left( \langle |P|^2 \rangle - \langle |P| \rangle^2 \right).$$

## 4. Polyakov loop distribution



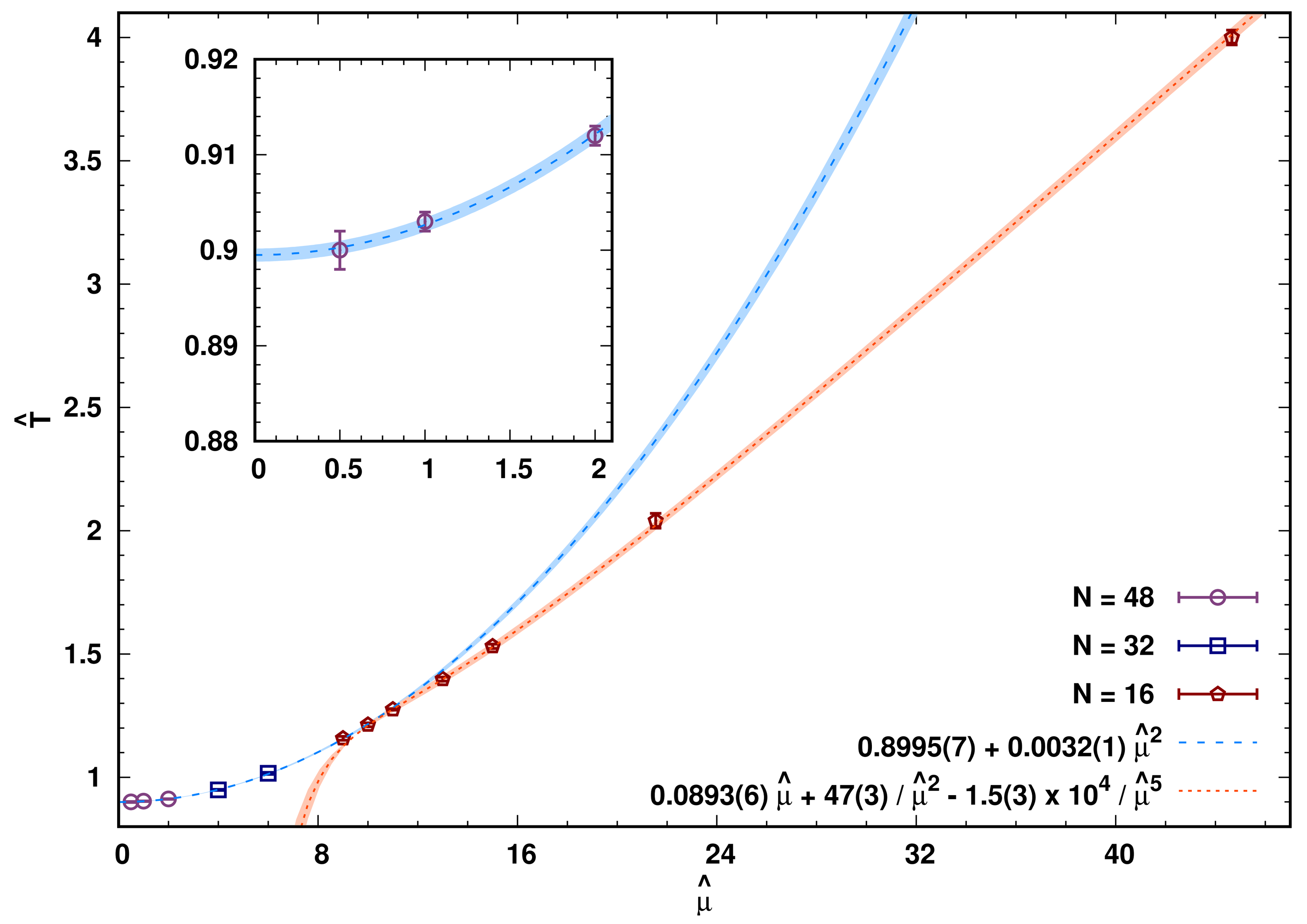

Polyakov loop scatter plots and its eigenvalue distributions for the configurations with $\widehat{\mu} = 2$, $N = 32$.

## 5. Signal and order of phase transition




The peak in the Polyakov loop susceptibility hints about phase transition in the model. Peak matching of susceptibility for different values of $N$ after normalization with $N^2$ indicates that the transition is of the first order. The configurations shown above are for $\widehat{\mu} = 4$.

## 6. Phase diagram




- Phase diagram smoothly interpolates between the limits of the bosonic BFSS model and the gauged Gaussian model.
- First order phase transition for all couplings (deformation values).
- $\widehat{\mu}_\star \sim 10$ separates the perturbative and non-perturbative regimes.

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- For future investigations, we are interested in exploring smaller $\widehat{\mu}$, which will require larger $N > 48$ to overcome challenges associated with flat directions and metastable vacua that make the numerical calculations more difficult.
- We also plan to study the 'ungauged' version of the BMN matrix model, with and without fermions.


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