Supersymmetric Theories on Lattice and Holography

apctp Seminar

Nov 23, 2022

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Outline

- Holographic motivation for studying theories non-perturbatively
- Supersymmetric theories and their lattice construction
- Phases in Maximal (and not maximal) supersymmetric Yang-Mills theories
- Future directions

Based on

arXiv:2109.01001 — *PoS **LATTICE21** (2022) 433*

arXiv:2201:08791 — *JHEP **05** (2022) 169*

arXiv:2212:xxxxx

with Raghav G. Jha (Jefferson Lab), Anosh Joseph (IISER Mohali), Abhishek Samlodia (Syracuse University), and David Schaich (University of Liverpool)

Gauge/Gravity Duality

Adv. Theor. Math. Phys. ***2*** *(1998) 231-252* *Maldacena*

4d $\mathcal{N} = 4$ SYM dual to Type *IIB* supergravity in decoupling limit

Maximally supersymmetric Yang-Mills (MSYM) theory in p+1 dimensions is dual to Dp-branes in supergravity at low temperatures in large N , strong coupling limit.

PRD ***58*** *(1998) 046004* *Itzhaki et al.*

Gauge/Gravity Duality

Gauge $\leftrightarrow$ Gravity

Strong $\leftrightarrow$ Weak

Hence, if we want to study this conjecture from field theory side, we need a non-perturbative setup.

LATTICE is one such non-perturbative alternative.

Non-perturbative information of String theory with help of AdS/CFT, Matrix Models

- 4d MSYM difficult to simulate using lattice setup as computationally costly.
- This talk will revolve around 1d and 2d theories, for which only a handful of lattice studies exist to probe duality.

github.com/daschaich/susy

SYM families

Lower dimensional SYM theories can be constructed by dimensionally reducing higher dimensional $\mathcal{N} = 1$ SYM theories

16 supersymmetries
Maximal SYM family

$d =$ [illegible]$, \mathcal{N} = 1$

$d =$ [illegible]$, \mathcal{N} = 2$

$\downarrow$

$d = 4, \mathcal{N} = 4$

$\downarrow$

$d = 3, \mathcal{N} = 8$

$\downarrow$

$d = 2, \mathcal{N} = (8, 8)$

$\downarrow$

$d = 1$, BFSS

$\downarrow$

$d = 0$, IKKT

8 supersymmetries — 4 supersymmetries
Non-Maximal SYM families

$d =$ [illegible]$, \mathcal{N} = 1$

$d =$ [illegible]$, \mathcal{N} = 2$

$\downarrow$

$d = 3, \mathcal{N} = 4$

$\downarrow$

$d = 2, \mathcal{N} = (4, 4)$

$d =$ [illegible]$, \mathcal{N} = 1$

$d =$ [illegible]$, \mathcal{N} = 2$

$\downarrow$

$d = 2, \mathcal{N} = (2, 2)$

Lattice construction using 'twisting' requires 2^d supersymmetries

SUSY on Lattice

SUSY algebra extension of Poincare algebra $\{Q, \overline{Q}\} \sim P_\mu$

P_μ ➜ generates infinitesimal translations ➜ Broken on lattice

Lattice studies of supersymmetric gauge theories

Recent review: *EPJ ST (2022)* *Schaich*

Though SUSY broken on lattice but we can preserve a subset of the algebra

SUSY theories discretized on lattice using "orbifolding" or "twisting" procedure

Phys.Rept. ***484*** *(2009) 71-130* *Catterall, Kaplan, Unsal*

SUSY on Lattice

In lattice simulations of supersymmetric theories slightly complicated

- Broken SUSY on lattice
- Duality check requires runs at large N, computationally expensive
- Flat directions ➜ $[X_i, X_j] = 0$ ➜ but scalar eigenvalues keeps on increasing because of access to continuum branch of the spectra
- Sign problem ➜ Boltzmann factor e^{-S} cannot be used as weight in stochastic process

JHEP **07** (2013) 101 Wiseman ⬅️ About these peculiar powers from SYM

For SYM theory in (1+p) dimensions

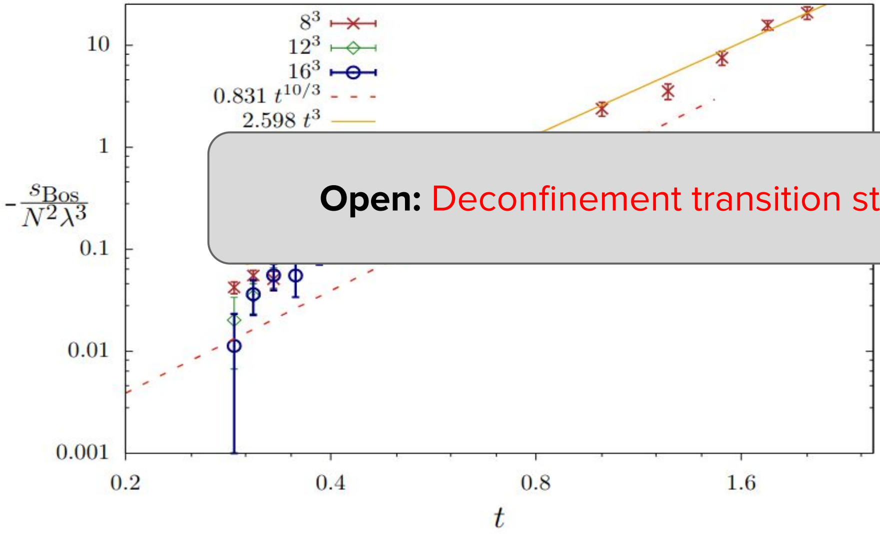
Bosonic action density $\propto t^{p+1}$, $t >> 1$

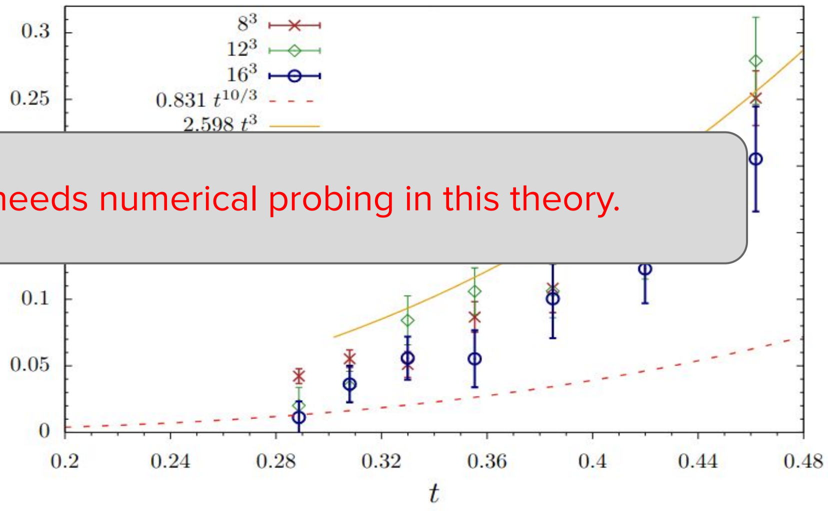
$\propto t^{(14-2p)/(5-p)}$, $t << 1$

Lattice Results

In conformal case both these cases are equivalent

p = 2





Open: Deconfinement transition still needs numerical probing in this theory.

PRD **102** (2020) 106009 Catterall, Giedt, Jha, Schaich, Wiseman

[JHEP 07 (2013) 101](#) Wiseman ⬅️ About these peculiar powers from SYM

For SYM theory in (1+p) dimensions

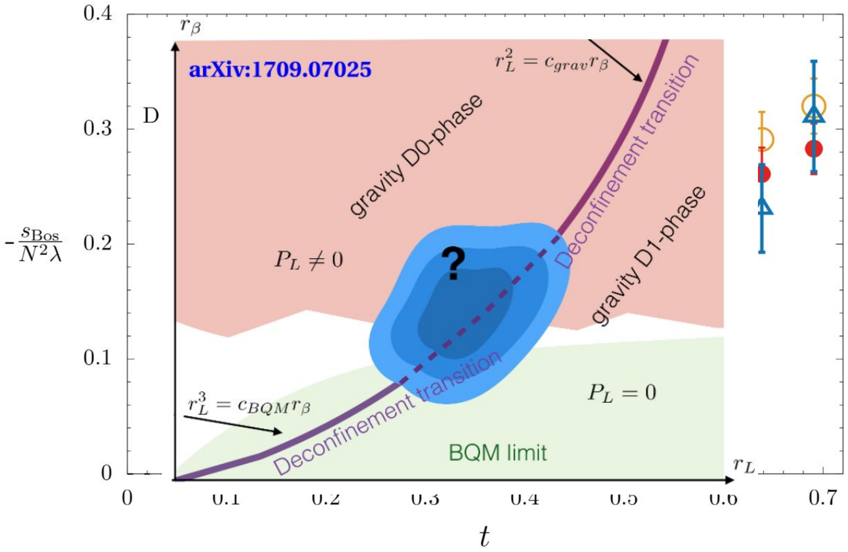
Bosonic action density $\propto t^{p+1}$, $t \gg 1$

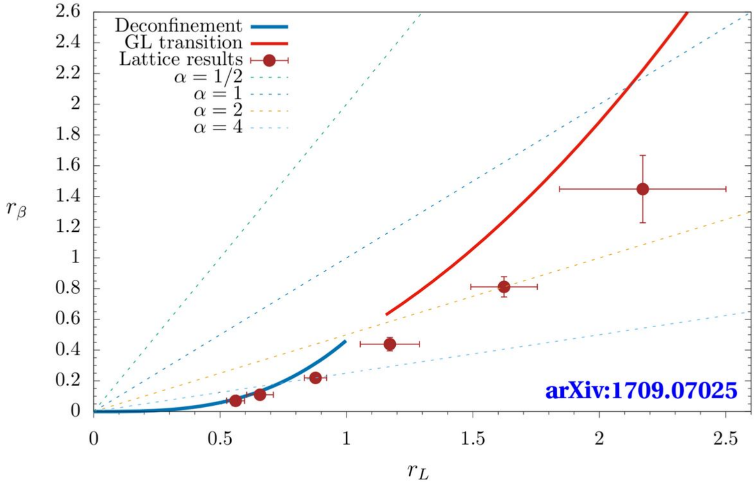
$\propto t^{(14-2p)/(5-p)}$, $t \ll 1$

Lattice Results

In conformal case both these cases are equivalent

p = 1





[PRD 97 (2018) 086020](#) Catterall, Jha, Schaich, Wiseman

2d $\mathcal{Q}$ = 4 SYM

$$\eta, \psi_a, \chi_{ab}$$

Fermions

- Obtained by dimensionally reducing $\mathcal{N} = 1$ SYM in 4d
- No holographic description

$$S = \frac{N}{4\lambda}\mathcal{Q}\int d^2x \ \mathrm{Tr}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left[\overline{\mathcal{D}}_a, \mathcal{D}_a\right] - \frac{1}{2}\eta d\right)$$

$\mathcal{F}_{ab} \to [\mathcal{D}_a, \mathcal{D}_b]$

$\mathcal{D}_a \to \partial_a + \mathcal{A}_a$

$\mathcal{A}_a \to A_a + iX_a$

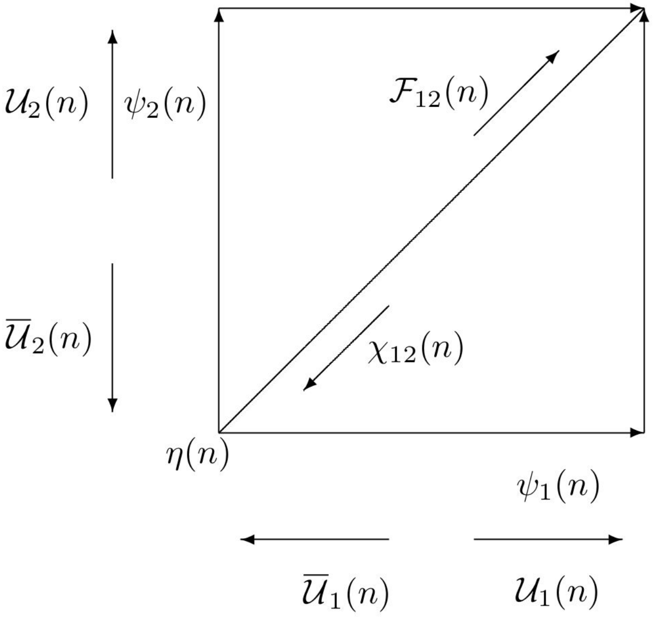
$$\mathcal{Q}\mathcal{A}_a = \psi_a, \qquad \mathcal{Q}\overline{\mathcal{A}}_a = 0, \qquad \mathcal{Q}\psi_a = 0,$$

$$\mathcal{Q}\chi_{ab} = -\overline{\mathcal{F}}_{ab}, \qquad \mathcal{Q}\eta = d, \qquad \mathcal{Q}d = 0.$$

After performing $\mathcal{Q}$ variation

2d $\mathcal{Q}$ = 4 SYM

$$S = \frac{N}{4\lambda}\int d^2x \, \mathrm{Tr}\left(-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2}\left[\overline{\mathcal{D}}_a, \mathcal{D}_a\right]^2 - \chi_{ab}\mathcal{D}_{[a}\,\psi_{\,b]} - \eta\overline{\mathcal{D}}_a\psi_a \right)$$



On Lattice

- Gauge field → Wilson link $\mathscr{A}_a(x) \rightarrow \mathscr{U}_a(n)$, on links of square lattice
- To preserve SUSY $\psi_a(n)$ lives on same links as bosonic superpartners
- η(n) associated with site
- χ_{ab} (n) lives on diagonal

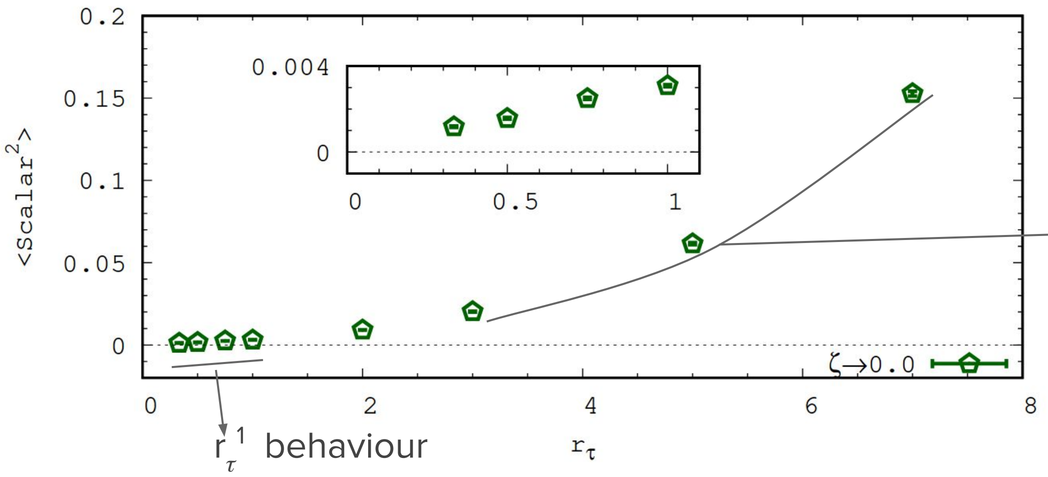
$$S = \frac{N}{4\lambda_{\mathrm{lat}}}\sum_n \mathrm{Tr}\Big[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) + \frac{1}{2}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n)\right)^2 -\chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{\,b]}(n) - \eta(n)\overline{\mathcal{D}}_a^{(-)}\psi_a(n)\Big],$$

Lattice Results

Scalar^2 → Tr (X^2)
24 x 24 lattice, N =12

To counter flat directions added scalar potential term $$\frac{N\mu^2}{4\lambda_{\text{lat}}}\sum_{n,a}\text{Tr}\left(\overline{\mathcal{U}}_a(n)\mathcal{U}_a(n)-\mathbb{I}_N\right)^2$$

Worked with different aspect ratios $\alpha \equiv \frac{r_x}{r_\tau} = \frac{N_x}{N_\tau}$ and deformation parameter $\mu = \zeta\frac{r_\tau}{N_\tau}$

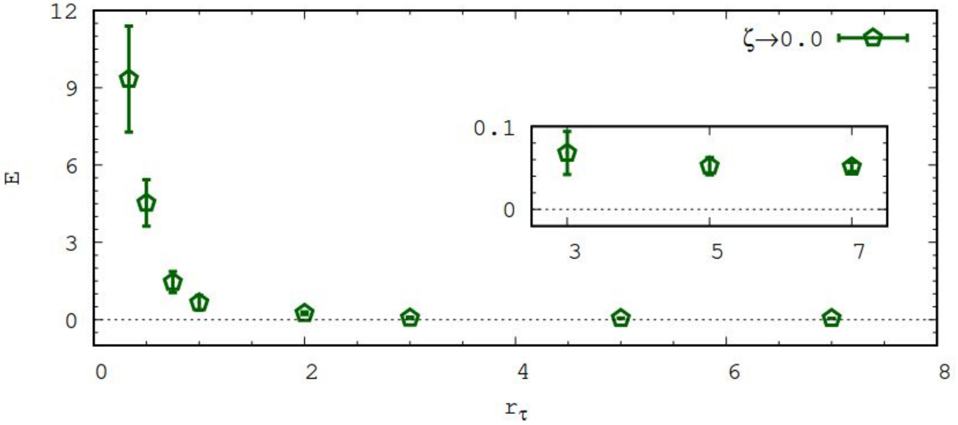


- Behaviour different than maximal cousin
- Existence of bound state at finite temperature

(To appear soon) **NSD**, Jha, Joseph, Schaich

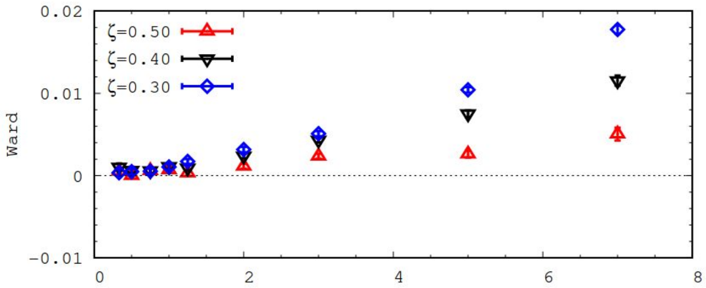
Lattice Results

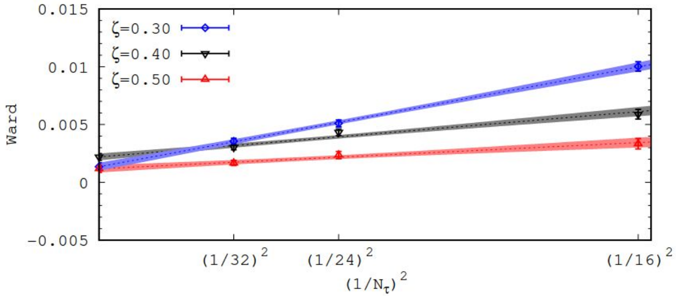
Preserved SUSY
24 x 24 lattice, N =12



$$E = \frac{3}{\lambda_{\text{lat}}}\left(1 - \frac{2}{3N^2} S_B\right)$$

$$\mathcal{Q} \sum_a \left(\eta \mathcal{U}_a \bar{\mathcal{U}}_a\right)$$



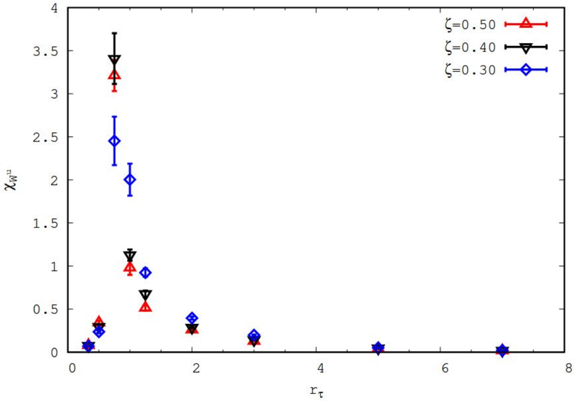


(To appear soon) **NSD**, Jha, Joseph, Schaich

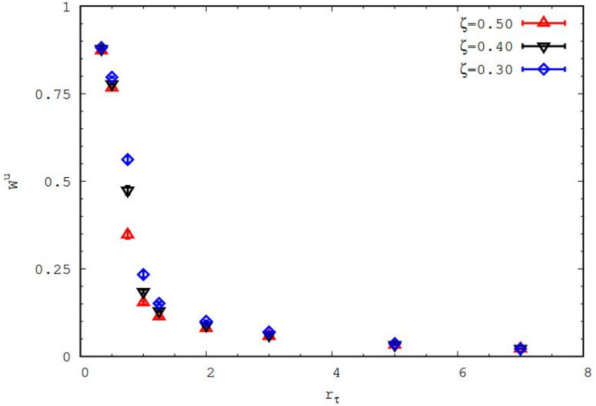
Lattice Results

Spatial deconfinement transition
24 x 24 lattice, N =12

Wilson loop along temporal and spatial direction

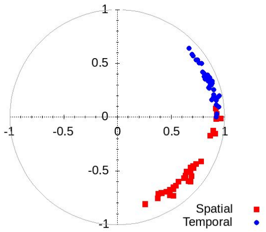


Variance of spatial WL

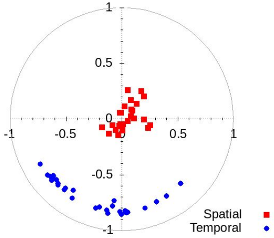


MC time history

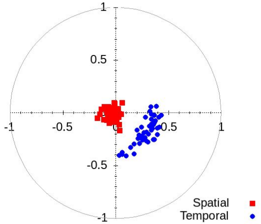
r_τ=0.5,ζ=0.3



r_τ=1.0,ζ=0.3



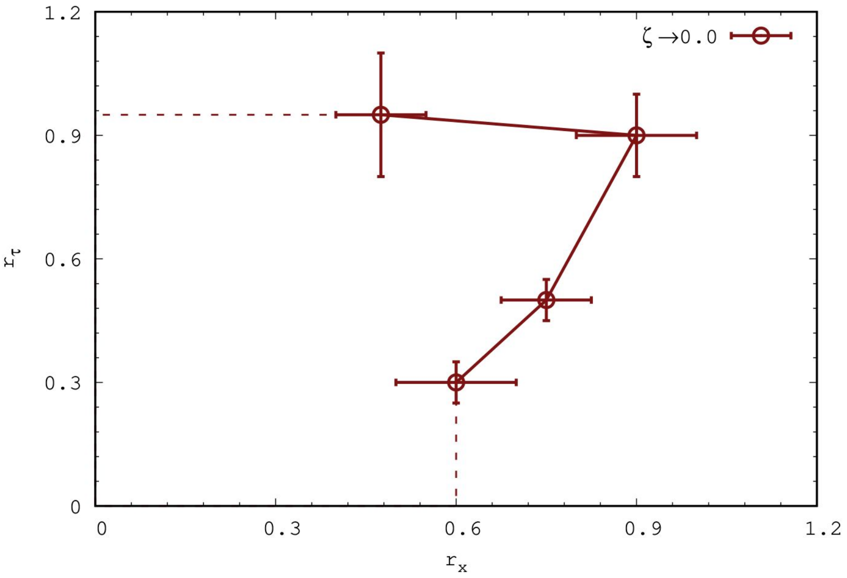
r_τ=3.0,ζ=0.3

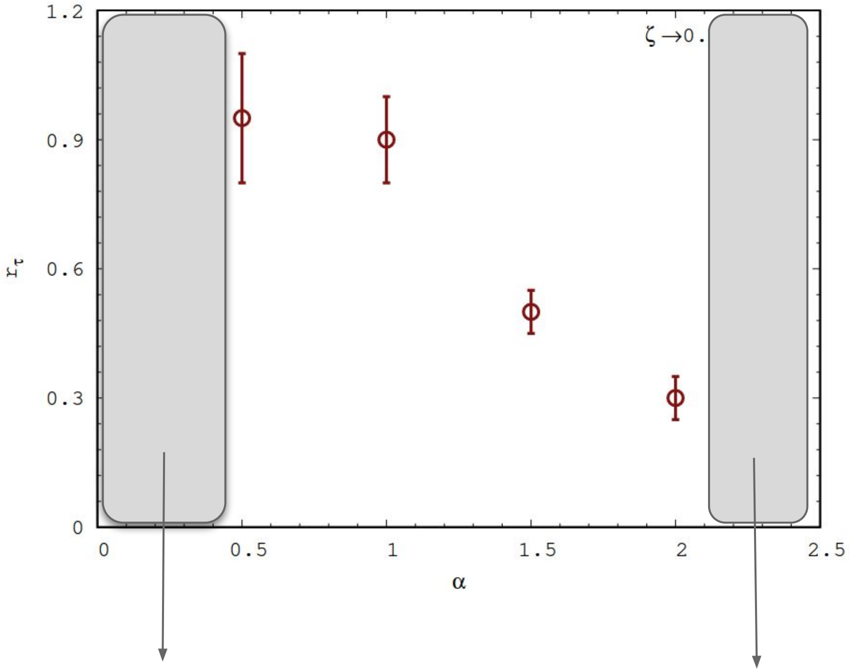


(To appear soon) **NSD**, Jha, Joseph, Schaich

Lattice Results

Phase diagram
Different aspect ratio **α**, N =12





Problematic regime in numerical simulations

(To appear soon) **NSD**, Jha, Joseph, Schaich

2d $\mathcal{Q} = 4$ SYM

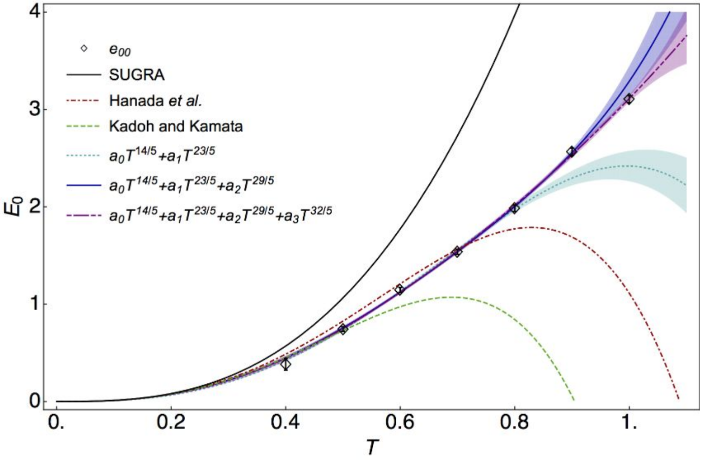
- Scalars show bound state behaviour
- Spatial deconfinement transition, but only limited to weak coupling regime
- Thermodynamics different than maximal counterpart
- More analysis required to probe if it admits holographic description : **Open**

Matrix Models

Back to Maximal theories

BFSS Model

$$S_{\mathrm{BFSS}} = \frac{N}{4\lambda}\int_0^\beta d\tau\ \mathrm{Tr}\Big\{ -(D_\tau X_i)^2 - \frac{1}{2}\sum_{i<j}[X_i, X_j]^2 + \Psi_\alpha^T \gamma^\tau_{\alpha\sigma} D_\tau \Psi_\sigma + \Psi_\alpha^T \gamma^i_{\alpha\sigma}[X_i, \Psi_\sigma]\Big\}$$



PRD **58** (2016) 094501 *Hanada et al.*

- SO(9) rotational symmetry

  A recent study using Gaussian expansion shows this symmetry broken like IKKT model

  arXiv:2209.01255 *Brahma, Brandenberger, Laliberte*

- Single deconfined phase in the theory

  A recent study with first results of confined phase

  JHEP **05** (2022) 096 *Bergner et al.*

BMN Model

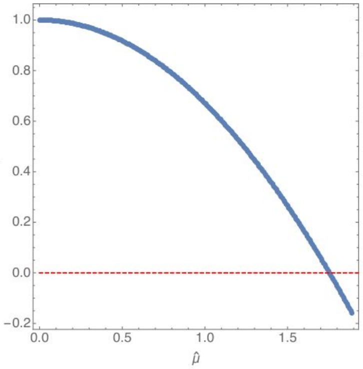
$$S_\mu = -\frac{N}{4\lambda}\int_0^\beta d\tau\ \mathrm{Tr}\left[\left(\frac{\mu}{3}X_I\right)^2 + \left(\frac{\mu}{6}X_A\right)^2 + \frac{\mu}{4}\Psi_\alpha^T\gamma_{\alpha\sigma}^{123}\Psi_\sigma - \frac{\sqrt{2}\mu}{3}\epsilon_{IJK}X_IX_JX_K\right]$$

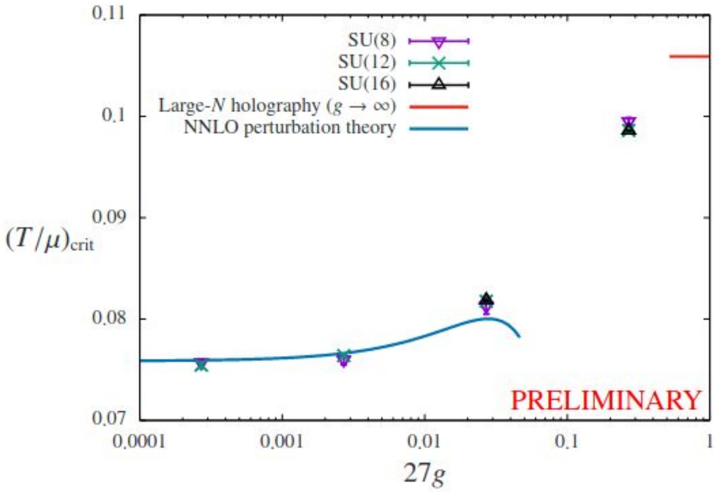
- Mass deformed version of BFSS
- SO(9) explicitly broken into SO(6) X SO(3)
- First order phase transition

Free energy of gravity solution

*JHEP **03** (2015) 069*

Costa, Greenspan, Penedones, Santos





Numerical simulated results

*PoS **LATTICE21** (2022) 433*

Schaich, Jha, Joseph

Open: Other thermodynamic properties ??

BMN Model

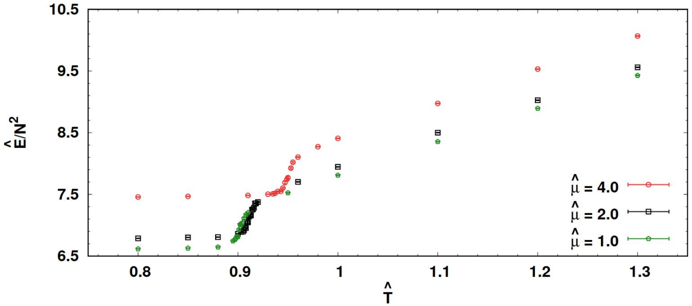
No fermions
➔ Clear deconfinement transition even in BFSS model

Easier to simulate
➔ Can work with large N setup

$$S_{\rm lat} = \frac{N}{4\lambda_{\rm lat}} \sum_{n=0}^{N_\tau - 1} \text{Tr}\Big[-(\mathcal{D}_+ X_i)^2 - \frac{1}{2}\sum_{i<j}[X_i, X_j]^2 - \left(\frac{\mu_{\rm lat}}{3} X_I\right)^2 - \left(\frac{\mu_{\rm lat}}{6} X_A\right)^2 + \frac{\sqrt{2}\mu_{\rm lat}}{3}\epsilon_{IJK} X_I X_J X_K \Big]$$

Our setup

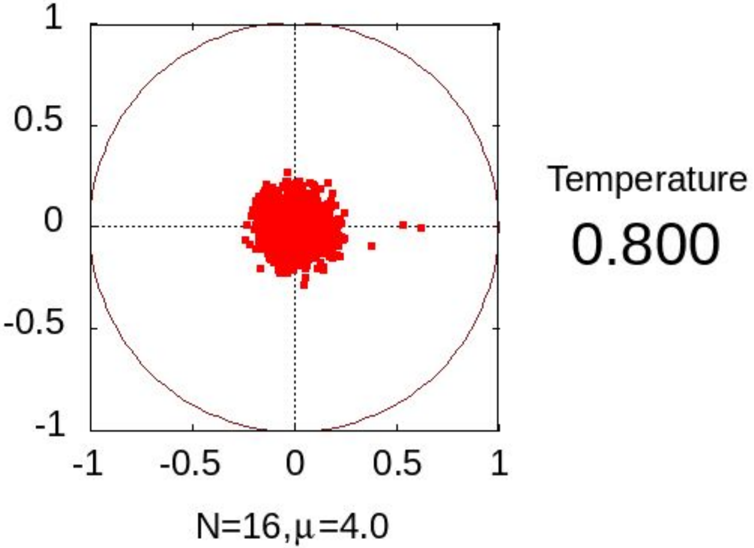
$$\frac{\widehat{E}}{N^2} \equiv \frac{E}{\lambda^{1/3}N^2} = \frac{1}{4N\lambda_{\rm lat}^{4/3}N_\tau}\left\langle \sum_{n=0}^{N_\tau-1} \text{Tr}\left(-\frac{3}{2}\sum_{i<j}[X_i, X_j]^2 - \frac{2\mu_{\rm lat}^2}{9}X_I^2 - \frac{\mu_{\rm lat}^2}{18}X_A^2 + \frac{5\sqrt{2}\mu_{\rm lat}}{6}\epsilon_{IJK}X^I X^J X^K \right)\right\rangle$$

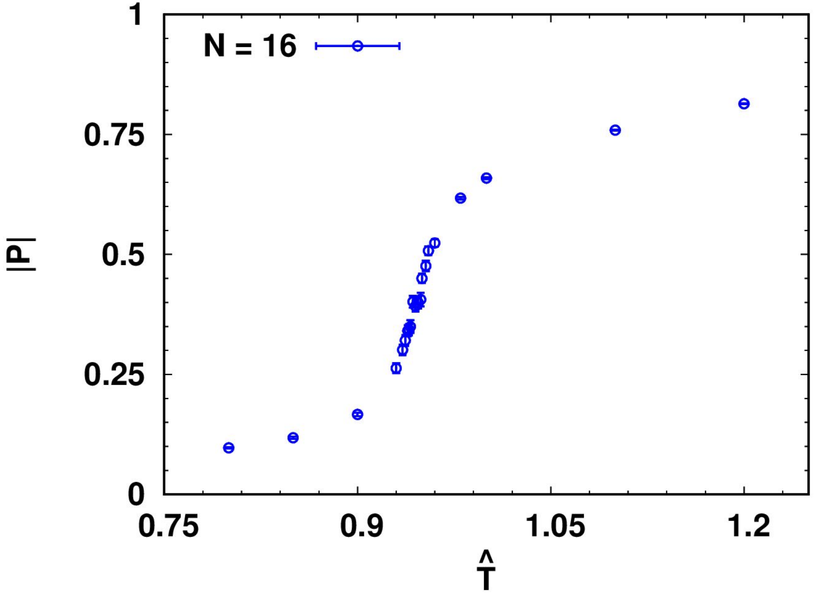


JHEP ***05*** *(2022) 169* **NSD**, Jha, Joseph, Samlodia, Schaich

Polyakov Loop

On lattice : $|P| = \left\langle \frac{1}{N} \left| \mathrm{Tr} \left(\prod_{n=0}^{N_\tau - 1} U(n) \right) \right| \right\rangle$





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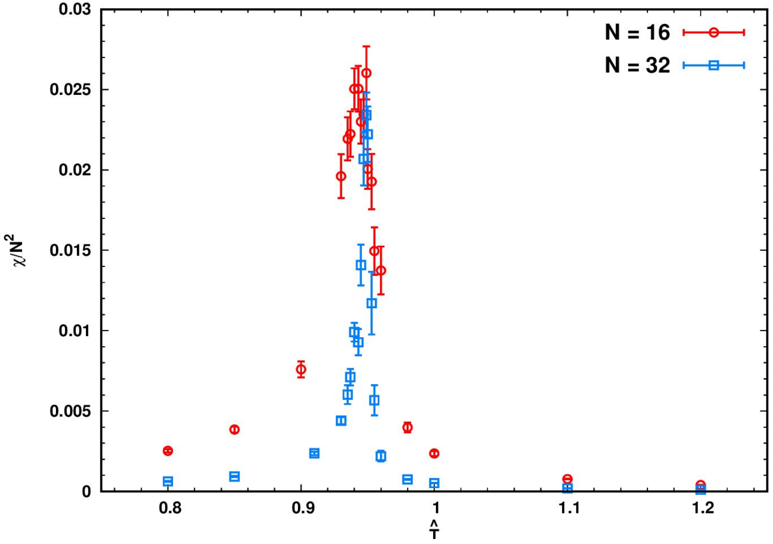
Transition Order

$$\chi \equiv N^2 \left(\langle |P|^2 \rangle - \langle |P| \rangle^2 \right)$$

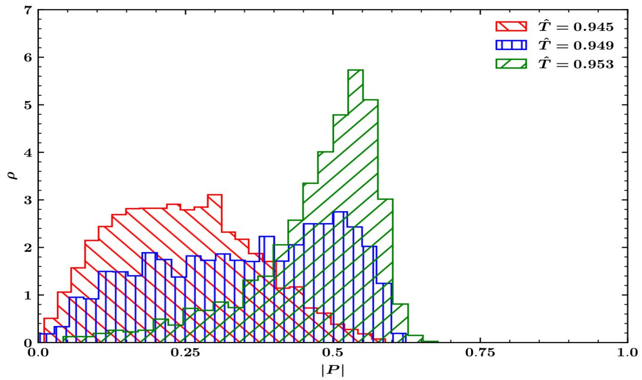


JHEP ***05*** *(2022) 169* **NSD**, Jha, Joseph, Samlodia, Schaich

- Susceptibility peaks at same height with N^2 normalization

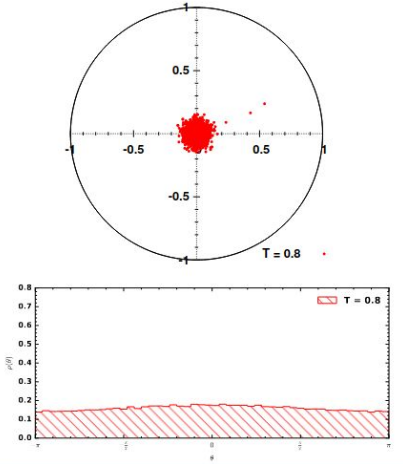
- First order phase transition *PRL* ***113*** *(2014) 091603*

Azuma, Morita, Takeuchi

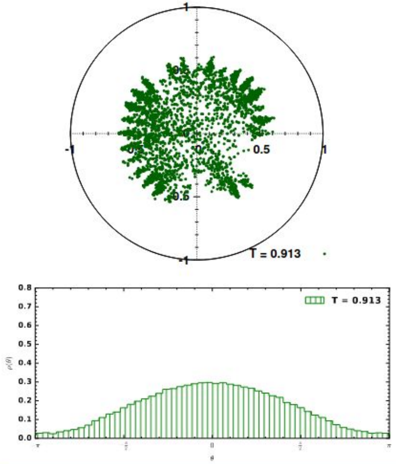


Different phases

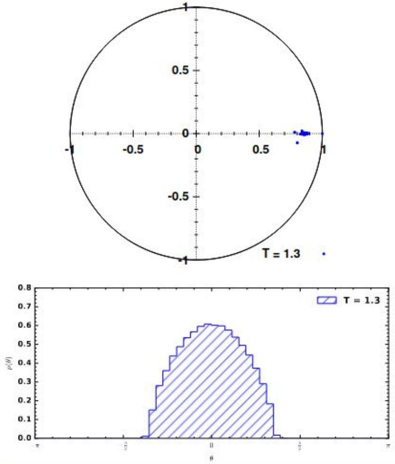
Angular distribution of Polyakov loop eigenvalues



$T = 0.8, \mu_{\text{lat}} = 2.0$
Uniform phase

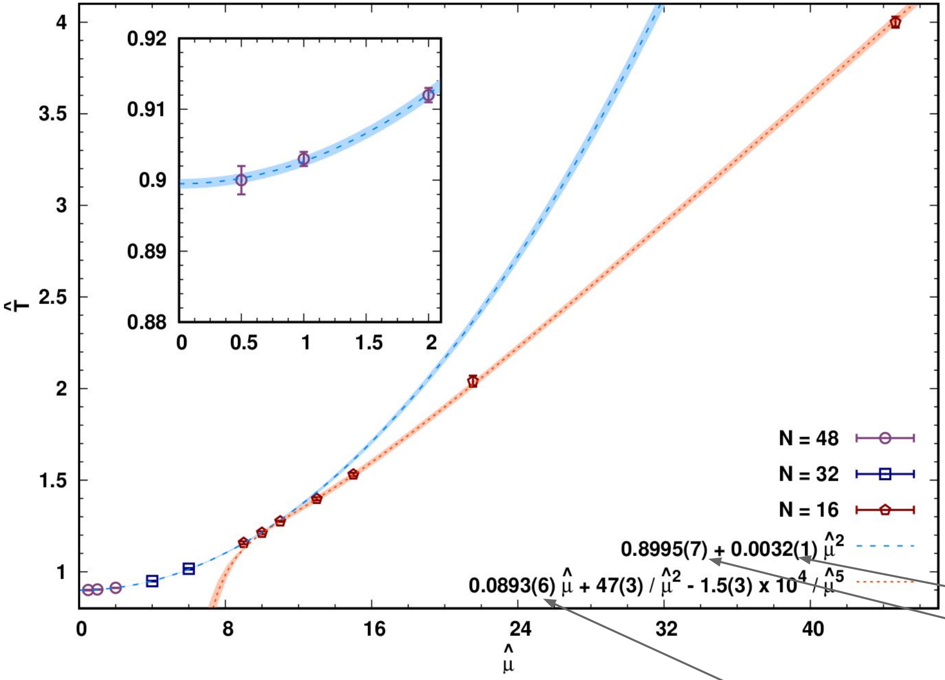


$T = 0.913, \mu_{\text{lat}} = 2.0$
Non-uniform phase



$T = 1.3, \mu_{\text{lat}} = 2.0$
Gapped phase

Phase Diagram



Perturbative calculation valid until $\mu \approx 10$, below it we enter strong coupling regime

First-order phase transition at all couplings

0.00330(2) *JHEP **05** (2022) 096*

0.8846(1) *Bergner et al.*

- Phase diagram smoothly interpolates between bosonic BFSS and gauged Gaussian limit

0.0893 *Adv.Theor.Math.Phys. **8** (2004) 603-696*

Aharony et al.

*JHEP **05** (2022) 169* **NSD**, Jha, Joseph, Samlodia, Schaich

Future Directions

- Numerical tools beyond Monte Carlo, especially for lower dimensional models
  - Numerical bootstrap is a viable option to investigate Matrix Models *JHEP **06** (2020) 090* *Lin*

- Numerically investigating non-gauge/gravity *JHEP **04** (2018) 084* *Maldacena, Milekhin*
  - Recent numerical results *JHEP **08** (2022) 178* *Pateloudis et al.*

- Continue exploring non-maximal supersymmetric theories

THANK YOU